The logic of Soft Computing IV

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Fuzzy Description Logics and the Semantic Web

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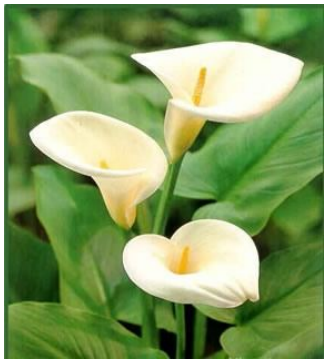


"Calla is a very large, long white flower on thick stalks"

Outline

- Introduction to Description Logics (DLs)
- Semantic Web, Ontologies and DLs
- Fuzzy DLs
- Conclusions & Future Work

Introduction to Description Logics

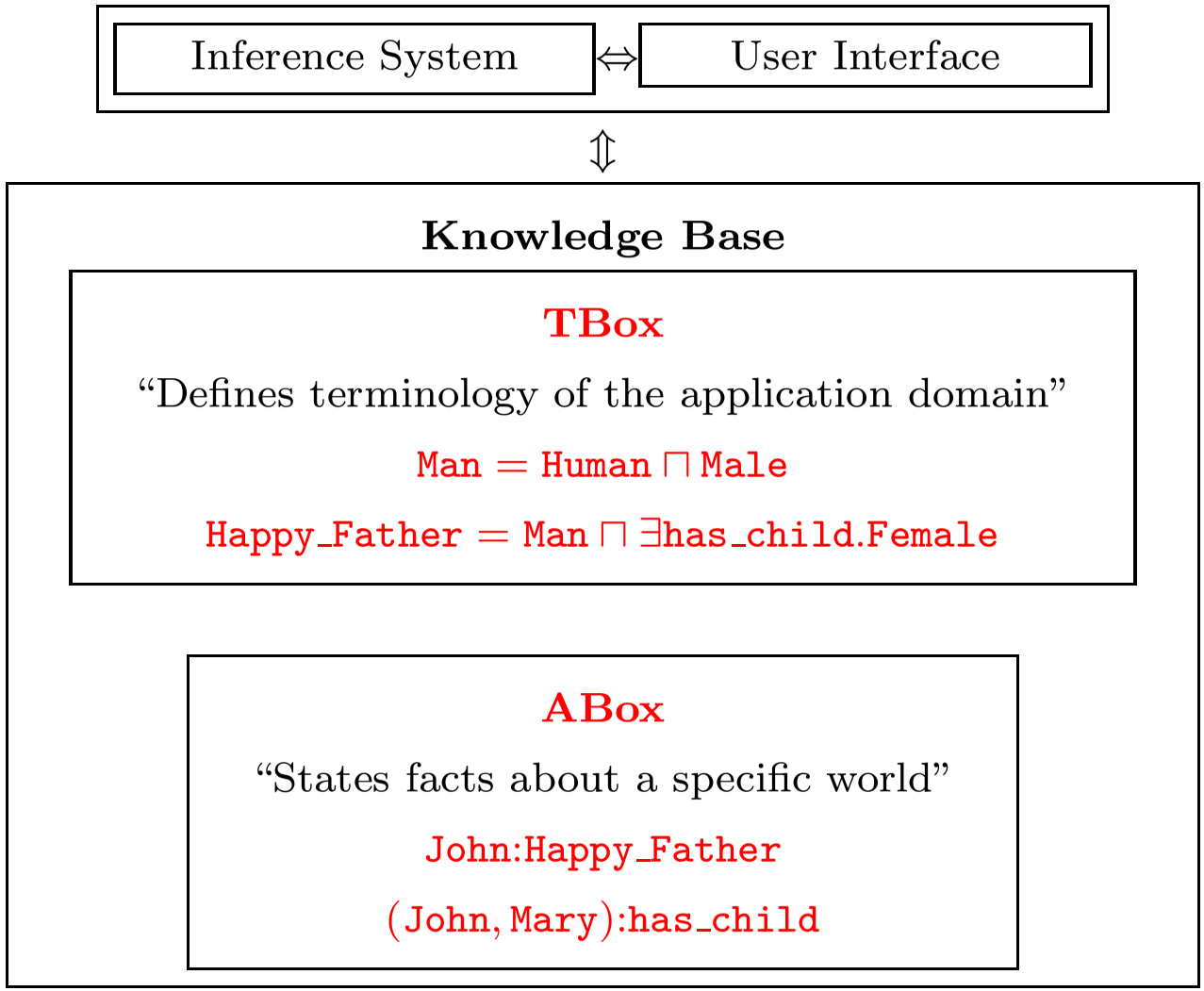
What Are Description Logics? (http://dl.kr.org/)

- A family of logic-based knowledge representation formalisms
  - Descendants of semantic networks and KL-ONE
  - Describe domain in terms of concepts (classes), roles (properties, relationships) and individuals
- Distinguished by:
  - Formal semantics (typically model theoretic)
    - Decidable fragments of FOL
    - Closely related to Propositional Modal & Dynamic Logics
    - Closely related to Guarded Fragment of FOL, L2 and C2
  - Provision of inference services
    - Decision procedures for key problems (e.g., satisfiability, subsumption)
    - Implemented systems (highly optimised)

DLs Basics

- Concept names are equivalent to unary predicates
  - In general, concepts equiv to formulae with one free variable
- Role names are equivalent to binary predicates
  - In general, roles equiv to formulae with two free variables
- Individual names are equivalent to constants
- Operators restricted so that:
  - Language is decidable and, if possible, of low complexity
  - No need for explicit use of variables
    - Restricted form of $\exists$ and $\forall$
  - Features such as counting can be succinctly expressed

Description Logic System



The DL Family

- A given DL is defined by set of concept and role forming operators
- Smallest propositionally closed DL is $\mathcal{ALC}$ ($\mathcal{A}$ttributive $\mathcal{L}$anguage with $\mathcal{C}$omplement)
  - Concepts constructed using $\sqcap, \sqcup, \neg, \exists$ and $\forall$

Syntax				*Example*
C, D	$\longrightarrow$	$\top$ |	(top concept)	
		$\bot$ |	(bottom concept)	
		A |	(atomic concept)	`Human`
		$C \sqcap D$ |	(concept conjunction)	`Human` $\sqcap$ `Male`
		$C \sqcup D$ |	(concept disjunction)	`Nice` $\sqcap$ `Rich`
		$\neg C$ |	(concept negation)	$\neg$`Meat`
		$\exists R.C$ |	(existential quantification)	$\exists$`has_child.Blond`
		$\forall R.C$	(universal quantification)	$\forall$`has_child.Human`

DL Semantics

- Semantics is given in terms of an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where
  - $\Delta^{\mathcal{I}}$ is the domain (a non-empty set)
  - $\cdot^{\mathcal{I}}$ is an interpretation function that maps:
    - Concept (class) name A into a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$
    - Role (property) name R into a subset $R^{\mathcal{I}}$ of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
    - Individual name a into an element of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
  - Interpretation function $\cdot^{\mathcal{I}}$ is extended to concept expressions:

$$
\begin{aligned}
\top^{\mathcal{I}} &= \Delta^{\mathcal{I}} \\
\bot^{\mathcal{I}} &= \emptyset \\
(C_1 \sqcap C_2)^{\mathcal{I}} &= {C_1}^{\mathcal{I}} \cap {C_2}^{\mathcal{I}} \\
(C_1 \sqcup C_2)^{\mathcal{I}} &= {C_1}^{\mathcal{I}} \cup {C_2}^{\mathcal{I}} \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
(\exists R.C)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \exists y.\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \forall y.\langle x, y\rangle \in R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}
\end{aligned}
$$

DLs and FOL

- $\mathcal{ALC}$ mapping to FOL: introduce
  - a unary predicate A for an atomic concept A
  - a binary predicate R for a role R
- Translate concepts C and D as follows

$$
\begin{array}{rcc}
t(\top, x) & = & \texttt{true} \\
t(\bot, x) & = & \texttt{false} \\
t(A, x) & \mapsto & A(x) \\
t(C_1 \sqcap C_2, x) & = & t(C_1, x) \wedge t(C_2, x) \\
t(C_1 \sqcup C_2, x) & \mapsto & t(C_1, x) \vee t(C_2, x) \\
t(\neg C, x) & = & \neg t(C, x) \\
t(\exists R.C, x) & = & \exists y.R(x, y) \wedge t(C, y) \\
t(\forall R.C, x) & = & \forall y.R(x, y) \Rightarrow t(C, y)
\end{array}
$$

DL Knowledge Base

- A DL Knowledge Base is a pair $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, where
  - $\mathcal{T}$ is a TBox containing general inclusion axioms of the form $C \sqsubseteq D$ ("concept D subsumes concept C"),

$$\mathcal{I} \models C \sqsubseteq D \quad \text{iff} \quad C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$$

    * concept definitions are of the form $A = C$ (equiv to $A \sqsubseteq C, C \sqsubseteq A$)
    * primitive concept definitions are of the form $A \sqsubseteq C$
    * Sometimes, a TBox can contain primitive and concept definitions only, where no atom can be defined more than once and no recursion is allowed $\mapsto$ Computational complexity changes dramatically
  - $\mathcal{A}$ is a ABox containing assertions of the form $a{:}C$ ("individual a is an instance of concept C) and $(a, b){:}R$ (individual b is an R-filler of individual a")

$$\mathcal{I} \models a{:}C \quad \text{iff} \quad a^{\mathcal{I}} \in C^{\mathcal{I}}$$
$$\mathcal{I} \models (a, b){:}R \quad \text{iff} \quad \langle a^{\mathcal{I}}, b^{\mathcal{I}} \rangle \in R^{\mathcal{I}}$$

Note on DL naming

$\mathcal{AL}$: $C, D \longrightarrow \top \mid \bot \mid A \mid C \sqcap D \mid \neg A \mid \exists R.\top \mid \forall R.C$

$\mathcal{C}$: Concept negation, $\neg C$. Thus, $\mathcal{ALC} = \mathcal{AL} + \mathcal{C}$

$\mathcal{S}$: Used for $\mathcal{ALC}$ with transitive roles $\mathcal{R}_+$

$\mathcal{U}$: Concept disjunction, $C_1 \sqcup C_2$

$\mathcal{E}$: Existential quantification, $\exists R.C$

$\mathcal{H}$: Role inclusion axioms, $R_1 \sqsubseteq R_2$

$\mathcal{N}$: Number restrictions, $(\geq n\ R)$ and $(\leq n\ R)$, e.g. $(\geq 3$ `has_Child`$)$ (has at least 3 children)

$\mathcal{Q}$: Qualified number restrictions, $(\geq n\ R.C)$ and $(\leq n\ R.C)$, e.g. $(\leq 2$ `has_Child.Adult`$)$ (has at most 2 adult children)

$\mathcal{O}$: Nominals (singleton class), $\{a\}$, e.g. $\exists$`has_child.{mary}`.
Note: $a{:}C$ equiv to $\{a\} \sqsubseteq C$ and $(a, b){:}R$ equiv to $\{a\} \sqsubseteq \exists R.\{b\}$

$\mathcal{I}$: Inverse role, R^-, e.g.

$\mathcal{F}$: Functional role, f

For instance,

$$\begin{aligned} \mathcal{SHIF} &= \mathcal{S} + \mathcal{H} + \mathcal{I} + \mathcal{F} = \mathcal{ALCR}_+\mathcal{HIF} \\ \mathcal{SHOIN} &= \mathcal{S} + \mathcal{H} + \mathcal{O} + \mathcal{I} + \mathcal{N} = \mathcal{ALCR}_+\mathcal{HOIN} \end{aligned}$$

Short History of Description Logics

Phase 1: Mostly system development (early eighties)

- Incomplete systems (Back, Classic, Loom, Kandor, ...)
- Based on structural algorithms

Phase 2: first tableaux algorithms and complexity results (mid-eighties - mid-nineties)

- Development of tableaux algorithms and complexity results
- Tableaux-based systems (Kris, Crack)
- Investigation of optimization techniques

Phase 3: Optimized systems for very expressive DLs (mid-nineties - . . .)

- Tableaux algorithms for very expressive DLs
- Highly optimised tableau systems (FaCT, DLP, Racer)
- Relationship to modal logic and decidable fragments of FOL

Latest developments

Phase 4:

- Mature implementations
- Mainstream applications and Tools
  - Databases
    - consistency of conceptual schema (EER, UML) using e.g. $\mathcal{DLR}$ (n-ary DL)
    - schema integration
    - Query subsumption (w.r.t. a conceptual schema)
  - Ontologies and the Semantic Web (and Grid)
    - Ontology engineering (design, maintenance, integration)
    - Reasoing with ontology-based markup (metadata)
    - Service description and discovery
  - Commercial implemenations
    - Cerebra, Racer

The Semantic Web

The Semantic Web Vision and DLs

- The WWW as we know it now
  - 1st generation web mostly handwritten HTML pages
  - 2nd generation (current) web often machine generated/active
  - Both intended for direct human processing/interaction
- In next generation web, resources should be more accessible to automated processes
  - To be achieved via semantic markup
  - Metadata annotations that describe content/function

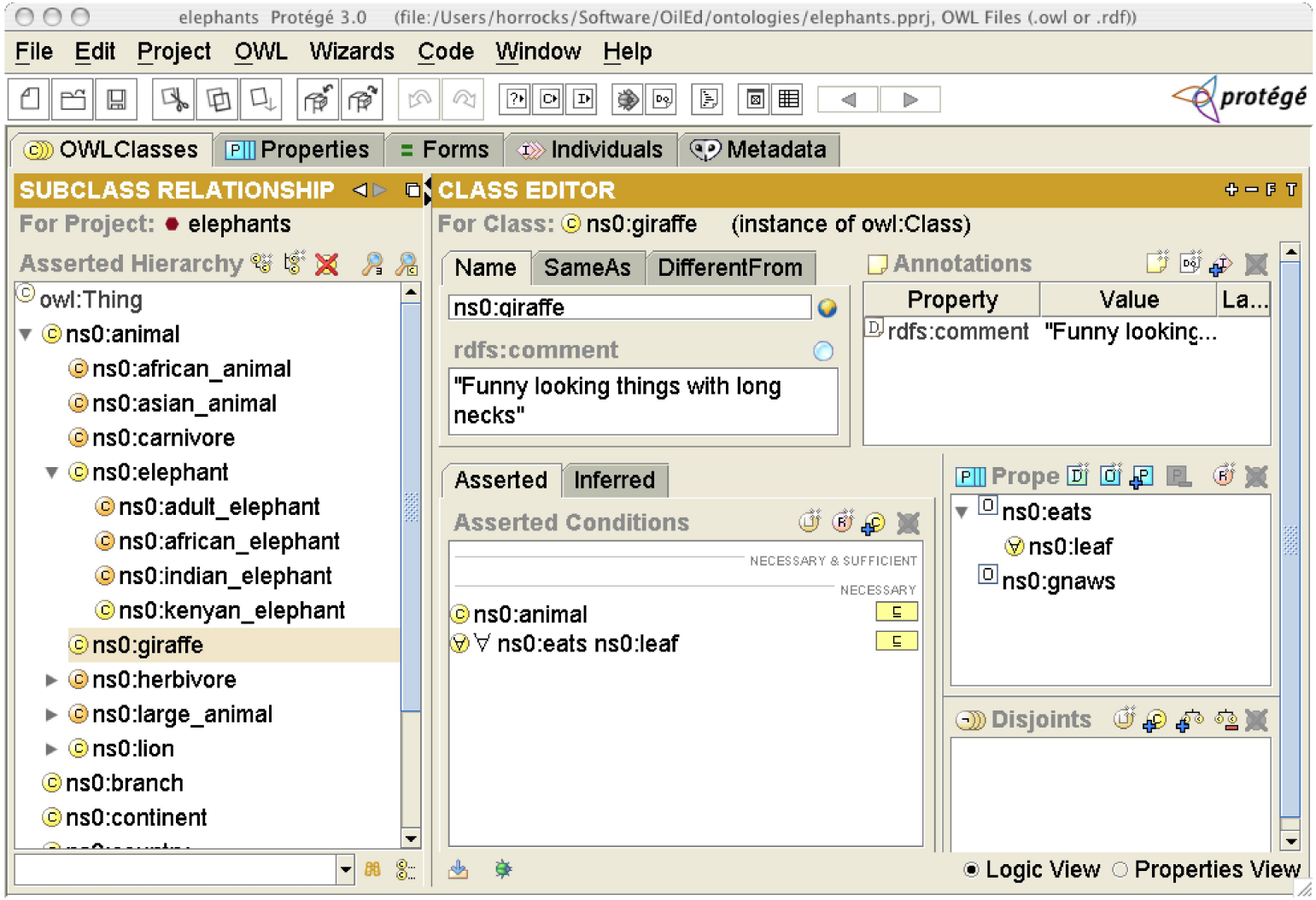
Ontologies

- Semantic markup must be meaningful to automated processes
- Ontologies will play a key role
  - Source of precisely defined terms (vocabulary)
  - Can be shared across applications (and humans)
- Ontology typically consists of:
  - Hierarchical description of important concepts in domain
  - Descriptions of properties of instances of each concept
- Ontologies can be used, e.g.
  - To facilitate agent-agent communication in e-commerce
  - In semantic based search
  - To provide richer service descriptions that can be more flexibly interpreted by intelligent agents

Example Ontology

- Vocabulary and meaning (definitions)
  - Elephant is a concept whose members are a kind of animal
  - Herbivore is a concept whose members are exactly those animals who eat only plants or parts of plants
  - Adult_Elephant is a concept whose members are exactly those elephants whose age is greater than 20 years
- Background knowledge/constraints on the domain (general axioms)
  - Adult_Elephants weigh at least 2,000 kg
  - All Elephants are either African_Elephants or Indian_Elephants
  - No individual can be both a Herbivore and a Carnivore

Example Ontology (Protégé)



Ontology Description Languages

- Should be sufficiently expressive to capture most useful aspects of domain knowledge representation
- Reasoning in it should be decidable and efficient
- Many different languages has been proposed: RDF, RDFS, OIL, DAML+OIL
- OWL (Ontology Web Language) is the current emerging language: it's a standard

OWL Language

- Three species of OWL
  - OWL full is union of OWL syntax and RDF (but, undecidable)
  - OWL DL restricted to FOL fragment (reasoning problem in NEXPTIME)
  - OWL Lite is easier to implement subset of OWL DL (reasoning problem in EXPTIME)
- Semantic layering
  - OWL DL $\equiv$ OWL full within Description Logic fragment
  - DL semantics officially definitive
- OWL DL based on $\mathcal{SHIQ}$ Description Logic ($\mathcal{ALCHIQR}_+$)
  - In fact it is equivalent to $\mathcal{SHOIN}(\mathtt{D})$
- OWL Lite based on $\mathcal{SHIF}$ Description Logic ($\mathcal{ALCHIFR}_+$)
- Benefits from many years of DL research
  - Formal properties well understood (complexity, decidability)
  - Known reasoning algorithms
  - Implemented systems (highly optimised)

Abstract Syntax	DL Syntax	Example
Descriptions (C)		
A (URI reference)	A	`Conference`
`owl:Thing`	$\top$	
`owl:Nothing`	$\bot$	
`intersectionOf`(C_1 C_2 ...)	$C_1 \sqcap C_2$	`Reference` $\sqcap$ `Journal`
`unionOf`(C_1 C_2 ...)	$C_1 \sqcup C_2$	`Organization` $\sqcup$ `Institution`
`complementOf`(C)	$\neg C$	$\neg$ `MasterThesis`
`oneOf`(o_1 ...)	$\{o_1, \ldots\}$	{"WISE","ISWC",...}
`restriction`(R `someValuesFrom`(C))	$\exists R.C$	$\exists$`parts.InCollection`
`restriction`(R `allValuesFrom`(C))	$\forall R.C$	$\forall$`date.Date`
`restriction`(R `hasValue`(o))	$R : o$	`date : 2005`
`restriction`(R `minCardinality`(n))	$(\geq n\ R)$	$\geqslant 1$ `location`
`restriction`(R `maxCardinality`(n))	$(\leq n\ R)$	$\leqslant 1$ `publisher`
`restriction`(U `someValuesFrom`(D))	$\exists U.D$	$\exists$`issue.integer`
`restriction`(U `allValuesFrom`(D))	$\forall U.D$	$\forall$`name.string`
`restriction`(U `hasValue`(v))	$U : v$	`series : "LNCS"`
`restriction`(U `minCardinality`(n))	$(\geq n\ U)$	$\geqslant 1$ `title`
`restriction`(U `maxCardinality`(n))	$(\leq n\ U)$	$\leqslant 1$ `author`

Abstract Syntax	DL Syntax	Example
Axioms		
Class(A partial $C_1 \ldots C_n$)	$A \sqsubseteq C_1 \sqcap \ldots \sqcap C_n$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
Class(A complete $C_1 \ldots C_n$)	$A = C_1 \sqcap \ldots \sqcap C_n$	Man $=$ Human $\sqcap$ Male
EnumeratedClass(A $o_1 \ldots o_n$)	$A = \{o_1\} \sqcup \ldots \sqcup \{o_n\}$	RGB $= \{\mathtt{r}\} \sqcup \{\mathtt{g}\} \sqcup \{\mathtt{b}\}$
SubClassOf($C_1 C_2$)	$C_1 \sqsubseteq C_2$	
EquivalentClasses($C_1 \ldots C_n$)	$C_1 = \ldots = C_n$	
DisjointClasses($C_1 \ldots C_n$)	$C_i \sqcap C_j = \bot, i \neq j$	Male $\sqsubseteq \neg$Female
ObjectProperty(R super $(R_1) \ldots$ super (R_n))	$R \sqsubseteq R_i$	HasDaughter $\sqsubseteq$ hasChild
domain(C_1) $\ldots$domain(C_n)	$(\geq 1\ R) \sqsubseteq C_i$	$(\geq 1$ hasChild$) \sqsubseteq$ Human
range(C_1) $\ldots$range(C_n)	$\top \sqsubseteq \forall R.D_i$	$\top \sqsubseteq \forall$hasChild.Human
[inverseof(R_0)]	$R = R_0^-$	hasChild $=$ hasParent$^-$
[symmetric]	$R = R^-$	similar $=$ similar$^-$
[functional]	$\top \sqsubseteq (\leq 1\ R)$	$\top \sqsubseteq (\leq 1$ hasMother$)$
[Inversefunctional]	$\top \sqsubseteq (\leq 1\ R^-)$	
[Transitive]	$Tr(R)$	$Tr($ancestor$)$
SubPropertyOf($R_1 R_2$)	$R_1 \sqsubseteq R_2$	
EquivalentProperties($R_1 \ldots R_n$)	$R_1 = \ldots = R_n$	cost $=$ price
AnnotationProperty(S)		

Abstract Syntax	DL Syntax	Example
DatatypeProperty(U super $(U_1) \ldots$ super (U_n))	$U \sqsubseteq U_i$	
domain(C_1) ...domain(C_n)	$(\geq 1\ U) \sqsubseteq C_i$	$(\geq 1$ hasAge$) \sqsubseteq$ Human
range(D_1) ...range(D_n)	$\top \sqsubseteq \forall U.D_i$	$\top \sqsubseteq \forall$hasAge.posInteger
[functional]	$\top \sqsubseteq (\leq 1\ U)$	$\top \sqsubseteq (\leq 1$ hasAge$)$
SubPropertyOf($U_1 U_2$)	$U_1 \sqsubseteq U_2$	hasName $\sqsubseteq$ hasFirstName
EquivalentProperties($U_1 \ldots U_n$)	$U_1 = \ldots = U_n$	
Individuals		
Individual(o type $(C_1) \ldots$ type (C_n))	o:C_i	tim:Human
value($R_1 o_1$) ...value($R_n o_n$)	(o, o_i):R_i	(tim, mary):hasChild
value($U_1 v_1$) ...value($U_n v_n$)	(o, v_1):U_i	(tim, 14):hasAge
SameIndividual($o_1 \ldots o_n$)	$o_1 = \ldots = o_n$	president_Bush = G.W.Bush
DifferentIndividuals($o_1 \ldots o_n$)	$o_i \neq o_j, i \neq j$	john $\neq$ peter

XML representation of OWL statements

E.g., $\texttt{Person} \sqcap \forall \texttt{hasChild}.(\texttt{Doctor} \sqcup \exists \texttt{hasChild}.\texttt{Doctor})$:

```
<owl:Class>
  <owl:intersectionOf rdf:parseType=" collection">
    <owl:Class rdf:about="#Person"/>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#hasChild"/>
      <owl:allValuesFrom>
        <owl:unionOf rdf:parseType=" collection">
          <owl:Class rdf:about="#Doctor"/>
          <owl:Restriction>
            <owl:onProperty rdf:resource="#hasChild"/>
            <owl:someValuesFrom rdf:resource="#Doctor"/>
          </owl:Restriction>
        </owl:unionOf>
      </owl:allValuesFrom>
    </owl:Restriction>
  </owl:intersectionOf>
</owl:Class>
```

Concrete domains in OWL

- OWL supports concrete domains: integers, strings, ...
- Clean separation between object classes and concrete domains
  - Disjoint interpretation domain: $d^{\mathcal{I}} \subseteq \Delta_D$, and $\Delta_D \cap \Delta^{\mathcal{I}} = \emptyset$
  - Disjoint concrete properties: $U^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta_D$, e.g., (`tim`, `14`):`hasAge`, (`sf`, "`SoftComputing`"):`hasAcronym`
- Philosophical reasons:
  - Concrete domains structured by built-in predicates
- Practical reasons:
  - Ontology language remains simple and compact
  - Semantic integrity of ontology language not compromised
  - Implementability not compromised can use hybrid reasoner
    - Only need sound and complete decision procedure for ${d_1}^{\mathcal{I}} \cap \ldots \cap {d_n}^{\mathcal{I}}$, where d_i is a (posssibly negated) datatype
- In the DL literature, these are called Concrete Domains
- Notation: $(\mathtt{D})$. E.g., $\mathcal{ALC}(\mathtt{D})$ is $\mathcal{ALC}$ + concrete domains, OWL DL = $\mathcal{SHOIN}(\mathtt{D})$

Reasoning with OWL DL

Reasoning

- What can we do with it?
  - Design and maintenance of ontologies
    - Check class consistency and compute class hierarchy
    - Particularly important with large ontologies/multiple authors ($\geq 2^10$ defined concepts)
  - Integration of ontologies
    - Assert inter-ontology relationships
    - Reasoner computes integrated class hierarchy/consistency
  - Querying class and instance data w.r.t. ontologies
    - Determine if set of facts are consistent w.r.t. ontologies
    - Determine if individuals are instances of ontology classes
    - Retrieve individuals/tuples satisfying a query expression
    - Check if one class subsumes (is more general than) another w.r.t. ontology
- How do we do it?
  - Use DLs reasoner (OWL DL = $\mathcal{SHOIN}(\mathtt{D})$)

Basic Inference Problems

Consistency: Check if knowledge is meaningful

- Is $\mathcal{K}$ consistent? $\mapsto$ There exists some model $\mathcal{I}$ of $\mathcal{K}$
- Is C consistent? $\mapsto C^{\mathcal{I}} \neq \emptyset$ for some some model $\mathcal{I}$ of $\mathcal{K}$

Subsumption: structure knowledge, compute taxonomy

- $\mathcal{K} \models C \sqsubseteq D$? $\mapsto C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all models $\mathcal{I}$ of $\mathcal{K}$

Equivalence: check if two classes denote same set of instances

- $\mathcal{K} \models C = D$? $\mapsto C^{\mathcal{I}} = D^{\mathcal{I}}$ for all models $\mathcal{I}$ of $\mathcal{K}$

Instantiation: check if individual a instance of class C

- $\mathcal{K} \models a{:}C$? $\mapsto a^{\mathcal{I}} \in C^{\mathcal{I}}$ for all models $\mathcal{I}$ of $\mathcal{K}$

Retrieval: retrieve set of individuals that instantiate C

- Compute the set $\{a : \mathcal{K} \models a{:}C\}$

Problems are all reducible to consistency, e.g.

- $\mathcal{K} \models C \sqsubseteq D$ iff $\langle \mathcal{T}, \mathcal{A} \cup \{a{:}C \sqcap \neg D\} \rangle$ not consistent
- $\mathcal{K} \models a{:}C$ iff $\langle \mathcal{T}, \mathcal{A} \cup \{a{:}\neg C\} \rangle$ not consistent

Reasoning in DLs: Basics

- Tableaux algorithm deciding concept consistency
- Try to build a tree-like model $\mathcal{I}$ of the input concept C
- Decompose C syntactically
  - Apply tableau expansion rules
  - Infer constraints on elements of model
- Tableau rules correspond to constructors in logic ($\sqcap, \sqcup, \ldots$)
  - Some rules are nondeterministic (e.g., $\sqcup, \leq$)
  - In practice, this means search
- Stop when no more rules applicable or clash occurs
  - Clash is an obvious contradiction, e.g., $A(x), \neg A(x)$
- Cycle check (blocking) may be needed for termination
- C satisfiable iff rules can be applied such that a fully expanded clash free tree is constructed

Tableaux checking consistency of an $\mathcal{ALC}$ concept

- Works on a tree (semantics through viewing tree as an ABox)
  - Nodes represent elements of $\Delta^{\mathcal{I}}$, labelled with sub-concepts of C
  - Edges represent role-successorships between elements of $\Delta^{\mathcal{I}}$
- Works on concepts in negation normal form: push negation inside using de Morgan' laws and

$$\begin{aligned} \neg(\exists R.C) &\mapsto \forall R.\neg C \\ \neg(\forall R.C) &\mapsto \exists R.\neg C \end{aligned}$$

- It is initialised with a tree consisting of a single (root) node x_0 with $\mathcal{L}(x_0) = \{C\}$
- A tree T contains a clash if, for a node x in T, $\{A, \neg A\} \subseteq \mathcal{L}(x)$
- Returns "C is consistent" if rules can be applied s.t. they yield a clash-free, complete (no more rules apply) tree

$\mathcal{ALC}$ Tableau rules

$x \bullet \{C_1 \sqcap C_2, \ldots\}$	$\longrightarrow_{\sqcap}$	$x \bullet \{C_1 \sqcap C_2, C_1, C_2, \ldots\}$
$x \bullet \{C_1 \sqcup C_2, \ldots\}$	$\longrightarrow_{\sqcup}$	$x \bullet \{C_1 \sqcup C_2, C, \ldots\}$ for $C \in \{C_1, C_2\}$
$x \bullet \{\exists R.C, \ldots\}$	$\longrightarrow_{\exists}$	$x \bullet \{\exists R.C, \ldots\}$ $R \downarrow$ $y \bullet \{C\}$
$x \bullet \{\forall R.C, \ldots\}$ $R \downarrow$ $y \bullet \{\ldots\}$	$\longrightarrow_{\forall}$	$x \bullet \{\exists R.C, \ldots\}$ $R \downarrow$ $y \bullet \{\ldots, C\}$

Soundness and Completeness

Theorem 1

1. *The tableau algorithm is a PSPACE (using depth-first search) decision procedure for consistency (and subsumption) of $\mathcal{ALC}$ concepts.*
2. *$\mathcal{ALC}$ has the tree-model property*

The tableau can be modified to a PSPACE decision procedure for

- e.g. $\mathcal{ALCN}$ and $\mathcal{ALCI}$
- TBox with acyclic concept definitions using lazy unfolding (unfolding on demand)
- Note: $\mathcal{ALC}$ with general inclusion axioms, $C \sqsubseteq D$, jumps to EXPTIME

Extensions: general inclusion axioms

- TBoxes with general inclusion axioms $C \sqsubseteq D$
  - Each node must be labeled with $\neg C \sqcup D$ (recall, $C \sqsubseteq D \equiv_{FOL} \forall x.\neg t(C,x) \vee t(D,x)$
  - However, termination not guaranteed

Given, $a{:}A$ and $A \sqsubseteq \exists R.A$ then

$a \bullet \{A, \neg A \sqcup \exists R.A\}$

$a \bullet \{A, \neg A \sqcup \exists R.A, \exists R.A\}$

$R \downarrow$

$y_1 \bullet \{A, \neg A \sqcup \exists R.A, \exists R.A\}$

$R \downarrow$

$y_2 \bullet \{A, \neg A \sqcup \exists R.A, \exists R.A\}$

$\vdots$

Note: y_1, y_2 share same properties

Blocking

- When creating new node, check ancestors for equal (superset) label
- If such a node is found, new node is blocked

Given, $a{:}A$ and $A \sqsubseteq \exists R.A$ then

$$a \bullet \{A, \neg A \sqcup \exists R.A, \exists R.A\}$$

$$R \downarrow$$

$$y_1 \bullet \{A, \neg A \sqcup \exists R.A, \exists R.A\} \text{ Node is blocked}$$

Note:

- Blocking may not work with more complex DLs (e.g., using $\mathcal{R}^-$)
- With number restrictions ($\mathcal{N}$), some satisfiable concepts have only non-finite models: e.g., testing $\neg C$ with $\mathcal{T} = \{\top \sqsubseteq \exists R.C, \top \sqsubseteq (\leq 1\ R^-)\}$
- Sophisticated methods has been developed to detect ∞ repetition of substructures

Focus of DL Research

- decidability/complexity of reasoning
- requires restricted description language
- system and complexity results available for various combinations of constructors

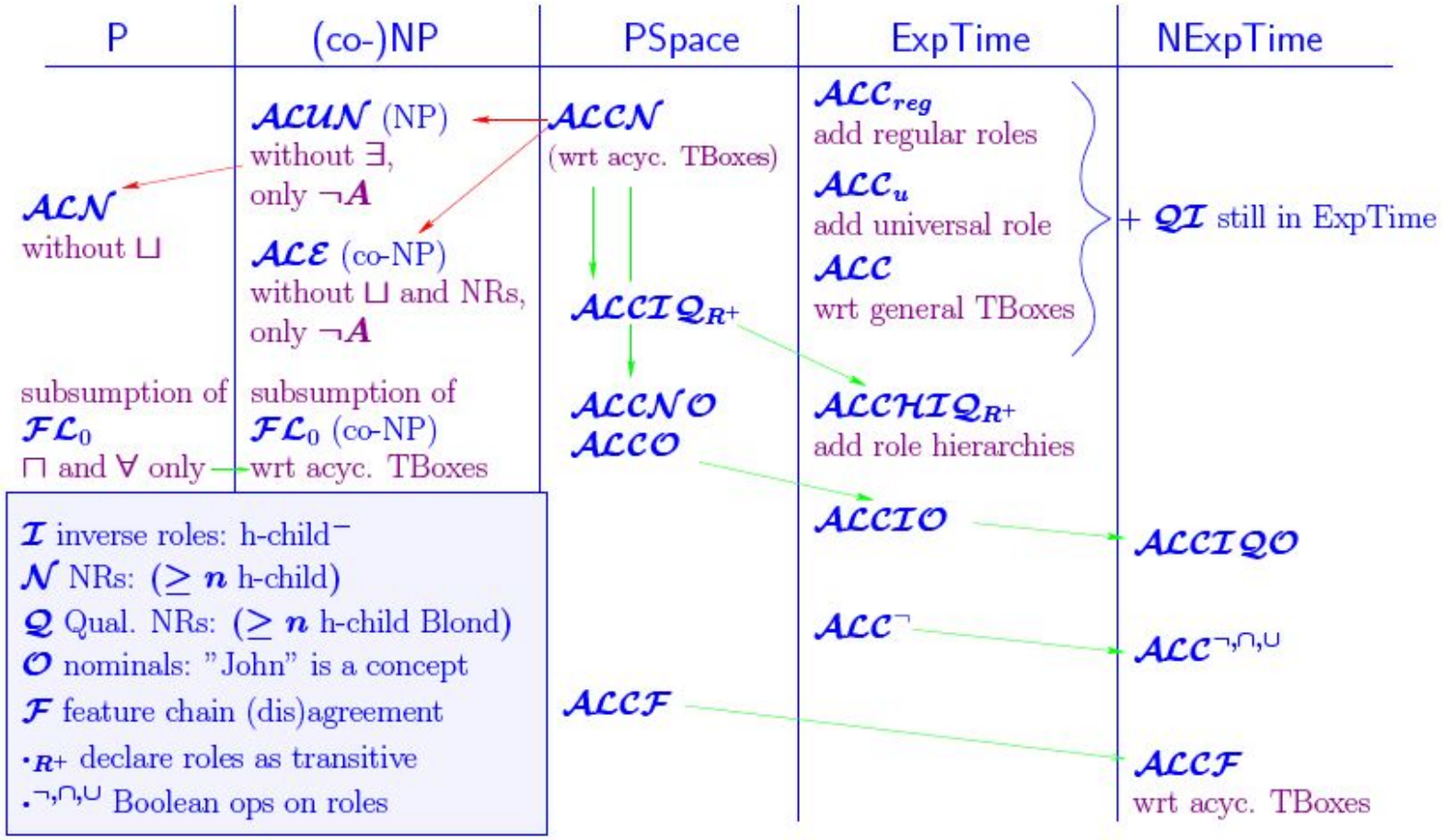
Reasoning feasible

$versus$

- application relevant concepts must be definable
- some applications domains require very expressive DLs
- efficient algorithms in practice for very expressive DLs

Expressivity sufficient

Some Complexity Results



Fuzzy Description Logics

"Calla is a very large, long white flower on thick stalks"

Objective

- To extend classical DLs towards the representation of and reasoning with vague concepts
- Application: Semantic Web
- Development of practical reasoning algorithms
- System implementations

Example (fuzzy DL-*Lite*, Current work)

$$
\begin{array}{rcl}
\texttt{Hotel} & \sqsubseteq & \exists \texttt{hasLocation} \\
\texttt{Conference} & \sqsubseteq & \exists \texttt{hasLocation} \\
\texttt{Hotel} & \sqsubseteq & \neg \texttt{Conference} \\
\texttt{Location}^{\mathcal{I}} & \subseteq & \texttt{GISCoordinates} \\
\texttt{distance}^{\mathcal{I}} & : & \texttt{GISCoord} \times \texttt{GISCoord} \rightarrow \mathbb{N} \\
 & & \mathit{distance}(x, y) = \ldots \\
\texttt{close}^{\mathcal{I}} & : & \mathbb{N} \rightarrow [0, 1] \\
 & & \mathit{close}(x) = \max(0, 1 - \frac{x}{1000})
\end{array}
$$

hasLocation	hasLocation	distance
hl1	cl1	300
hl1	cl2	500
hl2	cl1	750
hl2	cl2	750
⋮	⋮	

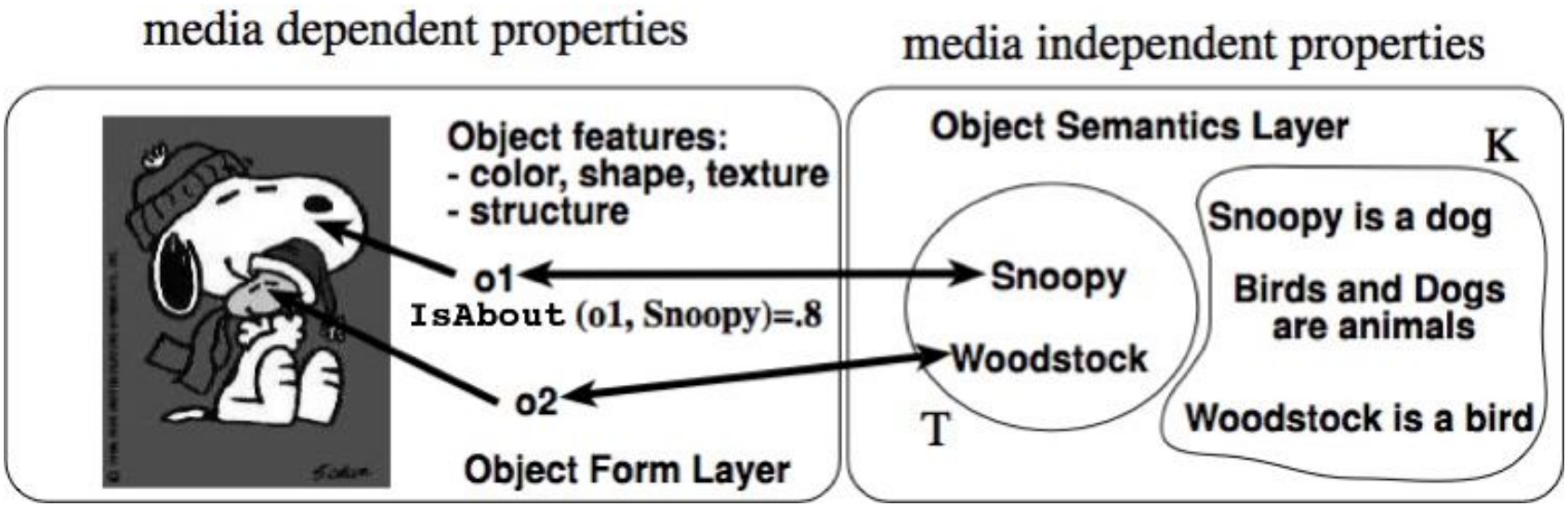
HotelID	hasLocation
h1	hl1
h2	hl2
⋮	⋮

ConferenceID	hasLocation
c1	cl1
c2	cl2
⋮	⋮

HotelID	closeness degree
h1	0.7
h2	0.25
⋮	⋮

"Find hotel close to conference c1": $\texttt{Query}(\texttt{c1}, h) \leftarrow$
$\texttt{Hotel}(h), \texttt{hasLocation}(h, hl), \texttt{Conference}(\texttt{c1}), \texttt{hasLocation}(\texttt{c1}, cl), \texttt{distance}(hl, cl, d), \texttt{close}(d)$

Example (Logic-based information retrieval model)



$$\begin{aligned} \texttt{Bird} &\sqsubseteq \texttt{Animal} \\ \texttt{Dog} &\sqsubseteq \texttt{Animal} \\ \texttt{snoopy} &: \texttt{Dog} \\ \texttt{woodstock} &: \texttt{Bird} \end{aligned}$$

ImageRegion	Object ID	isAbout degree
o1	snoopy	0.8
o2	woodstock	0.7
⋮	⋮	

$Query = \texttt{ImageRegion} \sqcap \exists \texttt{isAbout}.\texttt{Animal}$

$\texttt{Query}(ir) \leftarrow \texttt{ImageRegion}(ir), \texttt{isAbout}(ir, x), \texttt{Animal}(x)$

Example (Graded Entailment)

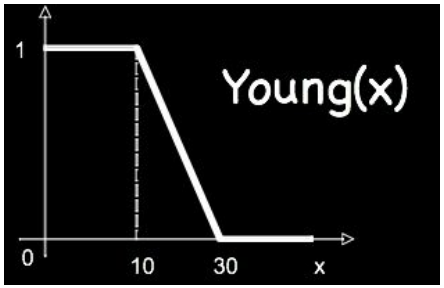
		
audi_tt	mg	ferrari_enzo

Car	speed
audi_tt	243
mg	$\leq$ 170
ferrari_enzo	$\geq$ 350

$$\texttt{SportsCar} \quad = \quad \texttt{Car} \sqcap \exists \texttt{hasSpeed.very(High)}$$

$$\mathcal{K} \quad \models \quad \langle \texttt{ferrari\_enzo:SportsCar}, 1 \rangle$$
$$\mathcal{K} \quad \models \quad \langle \texttt{audi\_tt:SportsCar}, 0.92 \rangle$$
$$\mathcal{K} \quad \models \quad \langle \texttt{audi\_tt:}\neg\texttt{SportsCar}, 0.72 \rangle$$

Example (Graded Subsumption)



$$\texttt{Minor} \quad = \quad \texttt{Person} \sqcap \exists \texttt{hasAge}. \leq_{18}$$

$$\texttt{YoungPerson} \quad = \quad \texttt{Person} \sqcap \exists \texttt{hasAge}.\texttt{Young}$$

$$\mathcal{K} \models \langle \texttt{Minor} \sqsubseteq \texttt{YoungPerson}, 0.2 \rangle$$

Note: without explicit membership function of `Young`, inference cannot be drawn

Basic principles of Fuzzy DLs

- In classical DLs, a concept C is interpreted by an interpretation $\mathcal{I}$ as a set of individuals
- In fuzzy DLs, a concept C is interpreted by $\mathcal{I}$ as a fuzzy set of individuals
- Each individual is instance of a concept to a degree in $[0, 1]$
- Each pair of individuals is instance of a role to a degree in $[0, 1]$

Fuzzy $\mathcal{ALC}$ concepts

Interpretation:

$$\begin{array}{lcl} \mathcal{I} & = & \Delta^{\mathcal{I}} \\ C^{\mathcal{I}} & : & \Delta^{\mathcal{I}} \to [0,1] \\ R^{\mathcal{I}} & : & \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1] \end{array} \qquad \begin{array}{lcl} t & = & \text{t-norm} \\ s & = & \text{s-norm} \\ n & = & \text{negation} \\ i & = & \text{implication} \end{array}$$

Concepts:

Syntax				*Semantics*		
C, D	$\longrightarrow$	$\top$	$\mid$	$\top^{\mathcal{I}}(x)$	$=$	1
		$\bot$	$\mid$	$\bot^{\mathcal{I}}(x)$	$=$	0
		A	$\mid$	$A^{\mathcal{I}}(x)$	$\in$	$[0,1]$
		$C \sqcap D$	$\mid$	$(C_1 \sqcap C_2)^{\mathcal{I}}(x)$	$=$	$t({C_1}^{\mathcal{I}}(x), {C_2}^{\mathcal{I}}(x))$
		$C \sqcup D$	$\mid$	$(C_1 \sqcup C_2)^{\mathcal{I}}(x)$	$=$	$s({C_1}^{\mathcal{I}}(x), {C_2}^{\mathcal{I}}(x))$
		$\neg C$	$\mid$	$(\neg C)^{\mathcal{I}}(x)$	$=$	$n(C^{\mathcal{I}}(x))$
		$\exists R.C$	$\mid$	$(\exists R.C)^{\mathcal{I}}(x)$	$=$	$\sup_{y \in \Delta^{\mathcal{I}}} t(R^{\mathcal{I}}(x,y), C^{\mathcal{I}}(y))$
		$\forall R.C$		$(\forall R.C)^{\mathcal{I}}(u)$	$=$	$\inf_{y \in \Delta^{\mathcal{I}}} i(R^{\mathcal{I}}(x,y), C^{\mathcal{I}}(y)\}$

Assertions: $\langle a{:}C, n\rangle$, $\mathcal{I} \models \langle a{:}C, n\rangle$ iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \geq n$ (similarly for roles)

- individual a is instance of concept C at least to degree n, $n \in [0,1] \cap \mathbb{Q}$

Inclusion axioms: $C \sqsubseteq D$,

- $\mathcal{I} \models C \sqsubseteq D$ iff $\forall x \in \Delta^{\mathcal{I}}.C^{\mathcal{I}}(x) \leq D^{\mathcal{I}}(x)$, (alternative, $\forall x \in \Delta^{\mathcal{I}}.i(C^{\mathcal{I}}(x), D^{\mathcal{I}}(x)) = 1$)

Basic Inference Problems

Consistency: Check if knowledge is meaningful

- Is $\mathcal{K}$ consistent?

Subsumption: structure knowledge, compute taxonomy

- $\mathcal{K} \models C \sqsubseteq D$?

Equivalence: check if two fuzzy concepts are the same

- $\mathcal{K} \models C = D$?

Graded instantiation: Check if individual a instance of class C to degree at least n

- $\mathcal{K} \models \langle a{:}C, n \rangle$?

BTVB: Best Truth Value Bound problem

- $glb(\mathcal{K}, a{:}C) = \sup\{n \mid \mathcal{K} \models \langle a{:}C, n \rangle\}$?

Retrieval: Rank set of individuals that instantiate C w.r.t. best truth value bound

- Rank the set $\mathcal{R}(\mathcal{K}, C) = \{\langle a, glb(\mathcal{K}, a{:}C) \rangle\}$

Some Notes on ...

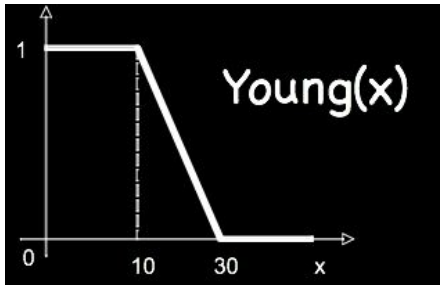
- Value restrictions:
  - In classical DLs, $\forall R.C \equiv \neg\exists R.\neg C$
  - The same is not true, in general, in fuzzy DLs (depends on the operators' semantics, not true in Gödel logic).
    $\forall \texttt{hasParent.Human} \not\equiv \neg\exists \texttt{hasParent}.\neg \texttt{Human}$??
- Models:
  - In classical DLs $\top \sqsubseteq \neg(\forall R.A) \sqcap (\neg\exists R.\neg A)$ has no classical model
  - In Gödel logic it has no finite model, but has an infinite model
- The choice of the appropriate semantics of the logical connectives is important.
  - Should have reasonable logical properties
  - Certainly it must have efficient algorithms solving basic inference problems

Towards fuzzy OWL Lite and OWL DL

- Recall that OWL Lite and OWL DL relate to $\mathcal{SHIF}(\mathtt{D})$ and $\mathcal{SHOIN}(\mathtt{D})$, respectively
- We need to extend the semantics of fuzzy $\mathcal{ALC}$ to fuzzy $\mathcal{SHOIN}(\mathtt{D}) = \mathcal{ALCR}_+\mathcal{HOINR}(\mathtt{D})$
- Additionally, we add modifiers (e.g., `very`)
- Additionally, we add concrete fuzzy concepts (e.g., `Young`)

Concrete fuzzy concepts

- E.g., $\mathtt{Small}, \mathtt{Young}, \mathtt{High}$, *etc.* with explicit membership function
- Use the idea of concrete domains:
  - $\mathtt{D} = \langle \Delta_{\mathtt{D}}, \Phi_{\mathtt{D}} \rangle$
  - $\Delta_{\mathtt{D}}$ is an interpretation domain
  - $\Phi_{\mathtt{D}}$ is the set of concrete fuzzy domain predicates d with a predefined arity n and fixed interpretation $d^{\mathtt{D}}: \Delta_{\mathtt{D}}^n \to [0,1]$
  - For instance,



$$\mathtt{Minor} \quad = \quad \mathtt{Person} \sqcap \exists \mathtt{hasAge}.{\leq_{18}}$$

$$\mathtt{YoungPerson} \quad = \quad \mathtt{Person} \sqcap \exists \mathtt{hasAge}.\mathtt{Young}$$

Modifiers

- Very, moreOrLess, slightly, etc.
- Apply to fuzzy sets to change their membership function
  - $\texttt{very}(x) = x^2$
  - $\texttt{slightly}(x) = \sqrt{x}$
- For instance,

$$\texttt{SportsCar} \quad = \quad \texttt{Car} \sqcap \exists \texttt{speed.very}(\texttt{High})$$

Number Restrictions

- May be a problem computationally
- The semantics of the concept $(\geq n\ S)$

$$(\geq n\ R)^{\mathcal{I}}(x) \quad = \quad \sup_{\{y_1,\ldots,y_n\}\subseteq\Delta^{\mathcal{I}}} \bigwedge_{i=1}^{n} R^{\mathcal{I}}(x,y_i)$$

- Is the result of viewing $(\geq n\ R)$ as the open first order formula

$$\exists y_1,\ldots,y_n.\ \bigwedge_{i=1}^{n} R(x,y_i) \wedge \bigwedge_{1\leq i<j\leq n} y_i \neq y_j\ .$$

- The semantics of the concept $(\leq n\ R)$

$$(\leq n\ R)^{\mathcal{I}}(x) = \neg(\geq n+1\ R)^{\mathcal{I}}(x)$$

- Note: $(\geq 1\ R) \equiv \exists R.\top$

Reasoning

- For full fuzzy $\mathcal{SHOIN}(\mathtt{D})$ or $\mathcal{SHIF}(\mathtt{D})$: does not exists yet!!
- Exists for fuzzy $\mathcal{ALC}(\mathtt{D})$ + modifiers + fuzzy concrete concepts (under Lukasiewicz semantics, also under "Zadeh semantics")
- Usual fuzzy tableaux calculus does not work anymore (problems with modifiers and concrete fuzzy concepts)
- Usual fuzzy tableaux calculus does not solve the BTVB problem
- New algorithm uses bounded Mixed Integer Programming oracle, as for Many Valued Logics
  - Recall: the *general MILP problem* is to find

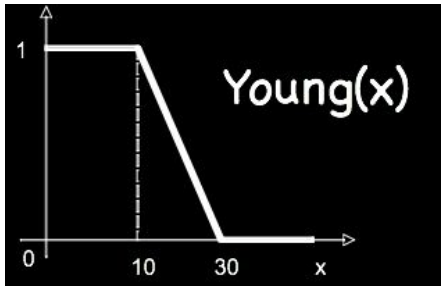
$$\bar{\mathbf{x}} \in \mathbb{Q}^k, \bar{\mathbf{y}} \in \mathbb{Z}^m$$

$$f(\bar{\mathbf{x}}, \bar{\mathbf{y}}) = \min\{f(\mathbf{x}, \mathbf{y}) \colon A\mathbf{x} + B\mathbf{y} \geq \mathbf{h}\}$$

A, B integer matrixes

Requirements

- Works for usual fuzzy DL semantics (Zadeh semantics) and Lukasiewicz logic
- Modifiers are definable as linear in-equations over $\mathbb{Q}, \mathbb{Z}$ (e.g., linear hedges)
- Fuzzy concrete concepts are definable as linear in-equations over $\mathbb{Q}, \mathbb{Z}$ (e.g., crisp, triangular, trapezoidal, left shoulder and right shoulder membership functions)



$$\begin{aligned} \texttt{Minor} &= \texttt{Person} \sqcap \exists \texttt{hasAge}. \leq_{18} \\ \texttt{YoungPerson} &= \texttt{Person} \sqcap \exists \texttt{hasAge}.\texttt{Young} \\ \texttt{Young} &= \texttt{ls}(10, 30) \end{aligned}$$

- Then

$$\begin{aligned} glb(\mathcal{K}, a{:}C) &= \min\{x \mid \mathcal{K} \cup \{\langle a{:}C \leq x\rangle \text{ satisfiable}\} \\ glb(\mathcal{K}, C \sqsubseteq D) &= \min\{x \mid \mathcal{K} \cup \{\langle a{:}C \sqcap \neg D \geq 1 - x\rangle \text{ satisfiable}\} \end{aligned}$$

  - Apply tableaux calculus, then use bounded Mixed Integer Programming oracle

$\mathcal{ALC}$ Tableau rules (excerpt)

$x \bullet \{\langle C_1 \sqcap C_2, \geq, l\rangle, \ldots\}$	$\longrightarrow_{\sqcap}$	$x \bullet \{\langle C_1 \sqcap C_2, \geq, l\rangle, \langle C_1, \geq, l\rangle, \langle C_2, \geq, l\rangle, \ldots\}$
$x \bullet \{\langle C_1 \sqcup C_2, \geq, l\rangle, \ldots\}$	$\longrightarrow_{\sqcup}$	$x \bullet \{\langle C_1 \sqcup C_2, \geq, l\rangle, \langle C_1, \geq, x_1\rangle, \langle C_2, \geq, x_2\rangle,$ $x_1 + x_2 = l, x_1 \leq y, x_2 \leq 1 - y,$ $x_i \in [0,1], y \in \{0,1\}, \ldots\}$
$x \bullet \{\langle \exists R.C, \geq, l\rangle, \ldots\}$	$\longrightarrow_{\exists}$	$x \bullet \{\langle \exists R.C, \geq, l\rangle, \ldots\}$ $\langle R, \geq, l\rangle \downarrow$ $y \bullet \{\langle C, \geq, l\rangle\}$
$x \bullet \{\langle \forall R.C, \geq, l_1\rangle, \ldots\}$ $\langle R, \geq, l_2\rangle \downarrow$ $y \bullet \{\ldots\}$	$\longrightarrow_{\forall}$	$x \bullet \{\langle \forall R.C, \geq, l_1\rangle, \ldots\}$ $\langle R, \geq, l_2\rangle \downarrow$ $y \bullet \{\ldots, \langle C, \geq, x\rangle$ $x + y \geq l_1, x \leq y, l_1 + l_2 \leq 2 - y,$ $x \in [0,1], y \in \{0,1\}\}$
$\vdots$	$\vdots$	$\vdots$
$x \bullet \{A \sqsubseteq C, \langle A, \geq, l\rangle, \ldots\}$	$\longrightarrow_{\sqsubseteq_1}$	$x \bullet \{A \sqsubseteq C, \langle C, \geq, l\rangle, \ldots\}$
$x \bullet \{C \sqsubseteq A, \langle A, \leq, l\rangle, \ldots\}$	$\longrightarrow_{\sqsubseteq_2}$	$x \bullet \{C \sqsubseteq A, \langle C, \leq, l\rangle, \ldots\}$
$\vdots$	$\vdots$	$\vdots$

Example

- Suppose

$$\mathcal{K} \quad = \quad \begin{cases} A \sqcap B \sqsubseteq C \\ \langle a{:}A \geq 0.3 \rangle \\ \langle a{:}B \geq 0.4 \rangle \end{cases}$$

$$Query \quad := \quad glb(\mathcal{K}, a{:}C) = \min\{x \mid \mathcal{K} \cup \{\langle a{:}C \leq x \rangle \text{ satisfiable}\}$$

Step	Tree	
1.	$a \bullet \{\langle A, \geq, 0.3\rangle, \langle B, \geq, 0.4\rangle, \langle C, \leq, x\rangle\}$	(Hypothesis)
2.	$\cup\{\langle A \sqcap B, \leq, x\rangle\}$	$(\rightarrow_{\sqsubseteq_2})$
3.	$\cup\{\langle A, \leq, x_1\rangle, \langle B, \leq, x_2\rangle\}$	$(\rightarrow_{\sqcap_{\leq}})$
	$\cup\{x = x_1 + x_2 - 1, 1 - y \leq x_1, y \leq x_2\}$	
	$\cup\{x_i \in [0,1], y \in \{0,1\}\}$	
4.	find $\min\{x \mid \langle a{:}A \geq 0.3\rangle, \langle a{:}B \geq 0.4\rangle,$	(MILP Oracle)
	$\langle a{:}C \leq x\rangle, \langle a{:}A \leq x_1\rangle, \langle a{:}B \leq x_2\rangle,$	
	$x = x_1 + x_2 - 1, 1 - y \leq x_1, y \leq x_2,$	
	$x_i \in [0,1], y \in \{0,1\}\}$	
5.	MILP oracle: $\mathbf{x = 0.3}$	

Implementation issues

- Several options exists:
  - Try to map fuzzy DLs to classical DLs
  - Try to map fuzzy DLs to some fuzzy/annotated logic programming framework
  - Build an ad-hoc theorem prover for fuzzy DLs, using e.g., MILP
- A theorem prover for fuzzy $\mathcal{ALC}$ + linear hedges + concrete fuzzy concepts, using MILP, has been implemented
  - Looking for volunteers to catch-up to the expressive power of fuzzy OWL Lite or fuzzy OWL DL (EXPTIME, NEXPTIME class)

Conclusions

- Classical DLs are the core of state of the art ontology description languages, as OWL DL and OWL Lite
- Efficient implementations exists, which take advantage of research on DLs decision algorithms and computational complexity analysis
- Fuzzy DLs aim at enhancing the expressive power of DLs towards the representation of vague concepts
- Research is still in it's infancy

Future Work

- To get rid with the subtleties of both Description Logics and Fuzzy Logics, experts from both areas are needed
- Research directions:
  - Computational complexity of the fuzzy DLs family
  - Design of efficient reasoning algorithms
  - Combining fuzzy DLs with Logic Programming
  - Language extensions: e.g. fuzzy quantifiers

    $$\texttt{TopCustomer} = \texttt{Customer} \sqcap (\texttt{Usually})\texttt{buys.ExpensiveItem}$$

    $$\texttt{ExpensiveItem} = \texttt{Item} \sqcap \exists \texttt{price.High}$$

  - Developing a system
  - ...