Representing Fuzzy Ontologies in OWL 2

Fernando Bobillo
fbobillo@unizar.es

Department of Computer Science and Systems Engineering
University of Zaragoza, Spain

Joint research with U. Straccia (ISTI-CNR, Pisa, Italy)

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Outline

1. Introduction

2. The Fuzzy DL \textit{SROIQ(D)}

3. Representing Fuzzy Ontologies using OWL 2

4. Related Work

5. Conclusions and Future Work
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1. Introduction

2. The Fuzzy DL $\textit{SROIQ(D)}$

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Motivation

- Fuzzy ontologies emerge as useful in several applications.
  - Several extension of Description Logics (DLs) can be found.
  - Some **fuzzy DL reasoners** have been implemented, such as FUZZYDL, DELOREAN, and FIRE.
  - Not surprisingly, each reasoner uses its own language for representing fuzzy ontologies and, thus, there is a need for a standard way to represent such information.

- In this work, as we do not expect a fuzzy OWL extension to become a W3C proposed standard in the near future, we identify the **syntactic differences** that a fuzzy ontology language has to cope with, and propose to use OWL 2 itself to represent them.

- More precisely, we use OWL 2 **annotation properties** to encode fuzzy SROIQ(D) ontologies, making it possible:
  - To use current OWL 2 editors for fuzzy ontology representation.
  - OWL 2 reasoners discard the fuzzy part of a fuzzy ontology, producing the same results as if would not exist.
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Fuzzy concrete domains. A pair $\langle \Delta_D, \Phi_D \rangle$, a concrete interpretation domain $\Delta_D$, and fuzzy concrete predicates $d \in \Phi_D$:

- $d \rightarrow \left\{ \begin{array}{ll}
\text{left}(k_1, k_2, a, b) & \text{(D1)} \\
\text{right}(k_1, k_2, a, b) & \text{(D2)} \\
\text{triangular}(k_1, k_2, a, b, c) & \text{(D3)} \\
\text{trapezoidal}(k_1, k_2, a, b, c, d) & \text{(D4)}
\end{array} \right.$

Fuzzy modifiers. A function $f_{mod} : [0, 1] \rightarrow [0, 1]$ applies to a fuzzy set to change its membership function:

- $mod \rightarrow \left\{ \begin{array}{ll}
\text{linear}(c) & \text{(M1)} \\
\text{triangular}(a, b, c) & \text{(M2)}
\end{array} \right.$
Fuzzy concepts

<table>
<thead>
<tr>
<th>Concepts</th>
<th>Syntax (C)</th>
<th>Semantics of $C^I(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C1) A</td>
<td>$A^I(x)$</td>
<td></td>
</tr>
<tr>
<td>(C2) ⊤</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(C3) ⊥</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(C4) $C \cap X D$</td>
<td>$C^I(x) \otimes_X D^I(x)$</td>
<td></td>
</tr>
<tr>
<td>(C5) $C \cup X D$</td>
<td>$C^I(x) \oplus_X D^I(x)$</td>
<td></td>
</tr>
<tr>
<td>(C6) $\neg_X C$</td>
<td>$\ominus_X C^I(x)$</td>
<td></td>
</tr>
<tr>
<td>(C7) $\forall_X R.C$</td>
<td>$\inf_{y \in \Delta_I} { R^I(x, y) \Rightarrow_X C^I(y) }$</td>
<td></td>
</tr>
<tr>
<td>(C8) $\exists_X R.C$</td>
<td>$\sup_{y \in \Delta_I} { R^I(x, y) \otimes_X C^I(y) }$</td>
<td></td>
</tr>
<tr>
<td>(C9) $\forall_X T.d$</td>
<td>$\inf_{v \in \Delta_D} { T^I(x, v) \Rightarrow_X d_D(v) }$</td>
<td></td>
</tr>
<tr>
<td>(C10) $\exists_X T.d$</td>
<td>$\sup_{v \in \Delta_D} { T^I(x, v) \otimes_X d_D(v) }$</td>
<td></td>
</tr>
<tr>
<td>(C11) ${ \alpha/a }$</td>
<td>$\alpha$ if $x = o^I_i$, 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>(C12) $\geq_X m S.C$</td>
<td>$\sup_{y_1, \ldots, y_m \in \Delta_I} (\min_{i=1}^{m} { S^I(x, y_i) \otimes_X C^I(y_i) }) \otimes_X (\otimes_{1 \leq j &lt; k \leq m} { y_j \neq y_k })$</td>
<td></td>
</tr>
<tr>
<td>(C13) $\leq_X n S.C$</td>
<td>$\inf_{y_1, \ldots, y_{n+1} \in \Delta_I} (\min_{i=1}^{n+1} { S^I(x, y_i) \otimes_X C^I(y_i) }) \Rightarrow_X (\otimes_{1 \leq j &lt; k \leq n+1} { y_j = y_k })$</td>
<td></td>
</tr>
<tr>
<td>(C14) $\geq_X m T.d$</td>
<td>$\sup_{v_1, \ldots, v_m \in \Delta_D} (\min_{i=1}^{m} { T^I(x, v_i) \otimes_X d_D(v_i) }) \otimes_X (\otimes_{j &lt; k} { v_j \neq v_k })$</td>
<td></td>
</tr>
<tr>
<td>(C15) $\leq_X n T.d$</td>
<td>$\inf_{v_1, \ldots, v_{n+1} \in \Delta_D} (\min_{i=1}^{n+1} { T^I(x, v_i) \otimes_X d_D(v_i) }) \Rightarrow_X (\otimes_{j &lt; k} { v_j = v_k })$</td>
<td></td>
</tr>
<tr>
<td>(C16) $\exists S.Self$</td>
<td>$S^I(x, x)$</td>
<td></td>
</tr>
<tr>
<td>(C17) $C \rightarrow_X D$</td>
<td>$C^I(x) \Rightarrow_X D^I(x)$</td>
<td></td>
</tr>
<tr>
<td>(C18) $mod(C)$</td>
<td>$f_{mod}(C^I(x))$</td>
<td></td>
</tr>
<tr>
<td>(C19) $[C \geq \alpha]$</td>
<td>$1$ if $C^I(x) \geq \alpha$, 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>(C20) $[C \leq \alpha]$</td>
<td>$1$ if $C^I(x) \leq \alpha$, 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>(C21) $\alpha \cdot C$</td>
<td>$\alpha \cdot C^I(x)$</td>
<td></td>
</tr>
</tbody>
</table>
Fuzzy roles and axioms

<table>
<thead>
<tr>
<th>Roles</th>
<th>Syntax ($R$)</th>
<th>Semantics of $R^I(x,y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R1)</td>
<td>$R_A$</td>
<td>$R^I_A(x,y)$</td>
</tr>
<tr>
<td>(R2)</td>
<td>$R^-$</td>
<td>$R^I(y,x)$</td>
</tr>
<tr>
<td>(R3)</td>
<td>$U$</td>
<td>1</td>
</tr>
<tr>
<td>(R4)</td>
<td>$mod(R)$</td>
<td>$f_{mod}(R^I(x,y))$</td>
</tr>
<tr>
<td>(R5)</td>
<td>$[R \geq \alpha]$</td>
<td>1 if $R^I(x,y) \geq \alpha$, 0 otherwise</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Axiom</th>
<th>Syntax ($\tau$)</th>
<th>Semantics ($\mathcal{I}$ satisfies $\tau$ if ...)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A1)</td>
<td>$\langle a : C \bowtie \alpha \rangle$</td>
<td>$C^I(a^I) \bowtie \alpha$</td>
</tr>
<tr>
<td>(A2)</td>
<td>$\langle (a, b) : R \bowtie \alpha \rangle$</td>
<td>$R^I(a^I, b^I) \bowtie \alpha$</td>
</tr>
<tr>
<td>(A3)</td>
<td>$\langle (a, b) : \neg X R \bowtie \alpha \rangle$</td>
<td>$\ominus X R^I(a^I, b^I) \bowtie \alpha$</td>
</tr>
<tr>
<td>(A4)</td>
<td>$\langle (a, v) : T \bowtie \alpha \rangle$</td>
<td>$T^I(a^I, v_D) \bowtie \alpha$</td>
</tr>
<tr>
<td>(A5)</td>
<td>$\langle (a, v) : \neg X T \bowtie \alpha \rangle$</td>
<td>$\ominus X T^I(a^I, v_D) \bowtie \alpha$</td>
</tr>
<tr>
<td>(A6)</td>
<td>$\langle a \neq b \rangle$</td>
<td>$a^I \neq b^I$</td>
</tr>
<tr>
<td>(A7)</td>
<td>$\langle a = b \rangle$</td>
<td>$a^I = b^I$</td>
</tr>
<tr>
<td>(A8)</td>
<td>$\langle C \subseteq X D \bowtie \alpha \rangle$</td>
<td>$\inf_{x \in \Delta^I} { C^I(x) \Rightarrow X D^I(x) } \bowtie \alpha$</td>
</tr>
<tr>
<td>(A9)</td>
<td>$\langle R_1 \ldots R_n \subseteq X R \bowtie \alpha \rangle$</td>
<td>$\inf_{x_1, x_{n+1} \in \Delta^I} { (\sup_{x_2 \ldots x_n \in \Delta^I} { (R_1^I(x_1, x_2) \otimes X \ldots \otimes X R_n^I(x_n, x_{n+1})) \Rightarrow X R^I(x_1, x_{n+1}) } ) \bowtie \alpha$</td>
</tr>
<tr>
<td>(A10)</td>
<td>$\langle T_1 \subseteq X T_2 \bowtie \alpha \rangle$</td>
<td>$\inf_{x \in \Delta^I, v \in \Delta^P} { T_1^I(x, v) \Rightarrow X T_2^I(x, v) } \bowtie \alpha$</td>
</tr>
<tr>
<td>(A11)</td>
<td>$\text{trans}_X(R)$</td>
<td>$\forall x, y, z \in \Delta^I, R^I(x, z) \otimes X R^I(z, y) \leq R^I(x, y)$</td>
</tr>
<tr>
<td>(A12)</td>
<td>$\text{dis}_X(S_1, S_2)$</td>
<td>$\forall x, y \in \Delta^I, S_1^I(x, y) \otimes X S_2^I(x, y) = 0$</td>
</tr>
<tr>
<td>(A13)</td>
<td>$\text{dis}_X(T_1, T_2)$</td>
<td>$\forall x \in \Delta^I, v \in \Delta^D, T_1^I(x, v) \otimes X T_2^I(x, v) = 0$</td>
</tr>
<tr>
<td>(A14)</td>
<td>$\text{ref}(R)$</td>
<td>$\forall x \in \Delta^I, R^I(x, x) = 1$</td>
</tr>
<tr>
<td>(A15)</td>
<td>$\text{irr}(S)$</td>
<td>$\forall x \in \Delta^I, S^I(x, x) = 0$</td>
</tr>
<tr>
<td>(A16)</td>
<td>$\text{sym}(R)$</td>
<td>$\forall x, y \in \Delta^I, R^I(x, y) = R^I(y, x)$</td>
</tr>
<tr>
<td>(A17)</td>
<td>$\text{asy}(S)$</td>
<td>$\forall x, y \in \Delta^I, \text{if } S^I(x, y) &gt; 0 \text{ then } S^I(y, x) = 0$</td>
</tr>
</tbody>
</table>
Definable elements

- **Definable concepts:**
  - **Weighted sum:** $(\alpha_1 \cdot C_1) \sqcup_L \cdots \sqcup_L (\alpha_k \cdot C_k)$.
  - **Fuzzy one-of:** $\{\alpha_1/o_1\} \sqcup_G \{\alpha_2/o_2\} \sqcup_G \cdots \sqcup_G \{\alpha_k/o_k\}$.

- **Definable axioms (given an R-implication):**
  - **Concept equivalence:** $\langle C_1 \sqsubseteq_X C_2 \geq 1 \rangle$ and $\langle C_2 \sqsubseteq_X C_1 \geq 1 \rangle$.
  - **Disjoint concepts:** $\langle C_1 \cap_X \cdots \cap_X C_n \sqsubseteq_X \bot \geq 1 \rangle$.
  - **Role domain:** $\langle \exists_X R.T \sqsubseteq_X C \geq 1 \rangle$.
  - **Role range:** $\langle T \sqsubseteq_X \forall_X R.C \geq 1 \rangle$.
  - **Role functionality:** $\langle T \sqsubseteq_X (\leq_X 1 R.T) \geq 1 \rangle$.

- **Syntactic sugar (not assumed for similarity with OWL 2):**
  - $\text{irr}(S) = T \sqsubseteq_X \neg \exists S.\text{Self}$
  - $\text{trans}(R) = RR \sqsubseteq_X R$
  - $\text{sym}(R) = R \sqsubseteq_X R^-$
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Intuitive idea

- The idea of our representation is to use an OWL 2 ontology, extending their elements with annotation properties of type `rdfs:comment`, representing the features of the fuzzy ontology that OWL 2 cannot directly encode.

Example

Consider the fuzzy concept assertion \( \langle \text{paul: Tall } \geq 0.5 \rangle \).
To represent it in OWL 2, we consider the crisp assertion `paul: Tall` as represented in OWL 2, i.e., `ClassAssertion(paul Tall)`.
Next, we add an annotation property stating \( \geq 0.5 \) to it.

- It is worth to note that OWL 2 only provides for annotations on ontologies, axioms, and entities.
- OWL DL is less expressive and only provides for annotations on ontologies and entities.
- For the sake of clarity, we will combine OWL 2 abstract syntax (for OWL 2), and an XML syntax (for annotation properties).
Syntactic Requirements of Fuzzy Ontologies

We will summarize the syntactic differences between the fuzzy and non-fuzzy ontologies. There are 8 cases which are non-exclusive (cases 3–5 can occur simultaneously, as well as cases 7–8).

1. **Fuzzy datatypes** do not have an equivalence in OWL 2: (D1–D4).
2. **Fuzzy modifiers** do not have an equivalence in OWL 2: (M1–M2).
3. Some **fuzzy concepts** require a **fuzzy logic**: (C4–C10), (C12–C15), (C17).
4. Some **fuzzy concepts** require a **degree** of truth: (C11), (C21).
5. Some **fuzzy concepts** do not have an equivalence in OWL 2: (C17)–(C21).
6. Some **fuzzy roles** do not have an equivalence in OWL 2: (R4–R5).
7. Some **axioms** require an inequality sign and a **degree** of truth: (A1–(A5), (A8)–(A10).
8. Some **axioms** require a **fuzzy logic**: (A3), (A5), (A8–A13).
1. Representing fuzzy datatypes

Example

Represent the age of a young person as \( \text{left}(0, 200, 10, 30) \). We use a datatype definition of base type \( \text{xsd:nonNegativeInteger} \) with range in \([0, 200] \):

\[
\text{DatatypeDefinition( YoungAge DatatypeRestriction(}
\quad \text{xsd:nonNegativeInteger}
\quad \text{xsd:minInclusive "0"^^xsd:integer}
\quad \text{xsd:maxInclusive "200"^^xsd:integer}
\) )
\]

Then we add the following annotation property to it:

\[
<\text{fuzzyOwl2 fuzzyType="datatype">}
\quad <\text{Datatype type="leftshoulder" a="10" b="30"} />
\)</fuzzyOwl2>

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2. Representing fuzzy modifiers

- Our fuzzy modifiers have parameters $a, b, c$.
- They can be represented as in the previous case, without representing `xsd:minInclusive` and `xsd:maxInclusive`.
- The value of `fuzzyType` will be `modifier`, and there will be a tag `Modifier` with an attribute `type` (possible values linear and triangular), and attributes $a, b, c$, depending on the type.

Example

We define the datatype `very`

```
DatatypeDefinition( very xsd:nonNegativeInteger )
```

Then, we add this annotation property to it:

```
<fuzzyOwl2 fuzzyType="modifier">
  <Modifier type="linear" c="0.8" />
</fuzzyOwl2>
```
3. Representing fuzzy concepts with a fuzzy logic

- An annotation in a named concept can specify the fuzzy logic.
  - Anonymous concept expressions must be explicitly named.
- The value of `fuzzyType` is `concept`.
- There is an optional tag `Logic` (possible values `goedel`, `lukasiewicz`, `product`, and `zadeh`). Default value: `goedel`.

Example (Concept `{1/germany} △_G {0.67/switzerland}`)

```xml
Class ( C Annotation ( rdfs:comment
  <fuzzyOwl2 fuzzyType="concept">
    <Logic>goedel</Logic>
  </fuzzyOwl2>
) )

EquivalentClasses( C ObjectUnionOf( Nom1 Nom2 ) )
```
4. Representing fuzzy concepts with a degree of truth

- An annotation in a named concept can specify the degree.
- Anonymous concept expressions must be explicitly named.
- The value of `fuzzyType` is `concept`.
- There is an optional tag `Degree` (with attribute `value`).

Example (Concepts \{1/germany\}, \{0.67/switzerland\})

```xml
Class ( Nom1 Annotation( rdfs:comment
    <fuzzyOwl2 fuzzyType="concept">
        <Degree value="1" />
    </fuzzyOwl2>
 )  ) Class ( Nom2 Annotation( rdfs:comment
    <fuzzyOwl2 fuzzyType="concept">
        <Degree value="0.67" />
    </fuzzyOwl2>
 )  ) EquivalentClasses( Nom1 ObjectOneOf ( germany ) )
EquivalentClasses( Nom2 ObjectOneOf ( switzerland ) )
```
5–6. Representing fuzzy concepts and roles without an equivalence in OWL

- We have **to create a new entity** (concept or role) denoting the elements, and to add an annotation property to it, describing the type of the constructor and the value of their parameters.

**Example (very(R))**

```xml
ObjectProperty ( veryR Annotation( rdfs:comment
  <fuzzyOwl2 fuzzyType="role">
    <Role type="modified" modifier="very" base="R" />
  </fuzzyOwl2>
) )
```
Some axioms may require a fuzzy logic, an inequality sign, or a degree of truth.

Similarly to cases 3–4, there are two optional tags:

- **Degree**, with attributes `value` and `sign` (possible values `geq`, `gre`, `leq`, and `les`),
- **Logic** (possible values `goedel`, `lukasiewicz`, `product`, or `zadeh`).

**Example**

Consider again the fuzzy concept assertion `<paul: Tall \geq 0.5>`. We extend the OWL 2 axiom `ClassAssertion(paul Tall)` with the following annotation property:

```xml
<fuzzyOwl2 fuzzyType="axiom">
  <Degree sign="geq" value="0.5" />
  <Logic>lukasiewicz</Logic>
</fuzzyOwl2>
```
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Related work

- An **OWL ontology** for fuzzy ontology representation using individuals to represent concepts, roles and axioms.
  - (Meta) logical problems, completely different and user-unfriendly way of modelling, and inefficient representation (it grows exponentially with the size of the ontology).

- **Fuzzy OWL and Fuzzy OWL 2.**
  - Obviously not complaint with OWL 2 and current ontology editors.

- Similar work covers just some of the cases:
  - A **pattern for uncertainty representation** in ontologies restricted to a subset of our case 7, axioms (A1).
  - **Probabilistic constraints** restricted to a subset of our case 7, axioms (A1) and (A8).

- **Crisp representations for fuzzy ontologies.**
  - Ok to reuse current DL reasoners, but not for modelling.
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Conclusions and future work

Our objective is **not to provide a standard** language for fuzzy ontology representation. This should involve the whole community.

We identified the **syntactical differences** of a fuzzy ontology language and provided a **representation using OWL 2**.

- A similar approach **cannot be represented in OWL DL** as it does not support rich enough annotation capabilities.

Our logic is very expressive, but it is **extensible** and can easily be augmented to support more fuzzy logics, fuzzy predicates . . .

**Methodology for fuzzy ontology development.**
- First, we can build the core part of the ontology as usual.
- Then, we add the fuzzy part with annotation properties.
- Non-fuzzy reasoners **discard the fuzzy part**.

**Parsers translating this representation into the syntax of some popular fuzzy DL reasoners** (**FUZZYDL** and **DELOREAN**).

**In future work**, we will develop a **graphical interface** (e.g. a Protégé plug-in) to make annotation properties transparent.
Comments?

Thank you very much for your attention