Fuzzy Bilateral Matchmaking in e-Marketplaces

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Outline

The Problem

- Suppose we have a buyer and a seller (agents)
  - the buyer describes what he is intended to buy and his preferences
  - the seller describes what he is intended to sell and his preferences
  - there is some background knowledge
- The objective is determine "an optimal" agreement among the two

Pareto optimal agreement: The utility of one agent cannot increase without decreasing the utility of the other

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  Pareto optimal agreement: The utility of one agent cannot increase without decreasing the utility of the other

Our Approach

- We present a logic-based approach
  - we use a logical language to express the seller's, the buyer's preferences and the background knowledge
- In particular, we use a Fuzzy Description Logic in order
  - to handle numerical features
  - to handle non numerical features
  - to deal with vagueness in buyer/seller preferences
- The language is part of the *fuzzyDL* system

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Running Example (buying/selling a car)

- A car seller sells a sedan car
- A buyer is looking for a second hand passenger car
- Both the buyer as well as the seller have preferences (constraints)
- Our aim is to find an optimal (Pareto) agreement

Running Example: the Background Knowledge

1. A sedan is a passenger car
2. A satellite alarm system is an alarm system
3. The navigator pack is a satellite alarm system with a GPS system
4. The Insurance Plus package is a driver insurance together with a theft insurance
5. The car colours are black or grey

Running Example: Buyer's preferences

1. He does not want to pay more than 30000 euro (buyer reservation value)
2. He wants a driver insurance and either a theft insurance or a fire insurance
3. He wants air conditioning and the external colour should be either black or grey
4. Preferably the price is no more than 25000 euro, but he can go up to 30000 euro to a lesser degree of satisfaction
5. The kilometer warranty is preferrably at least 180000 km, but he may go down to 120000 km to a lesser degree of satisfaction
6. Some preferences are more important than others

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Running Example: Seller's preferences

1. He wants to sell no less than 22000 euro (seller reservation value)
2. Preferably the seller sells the Insurance Plus package
3. The kilometer warranty is preferrably at most 140000 km, but he may go up to 175000 km to a lesser degree of satisfaction
4. Preferably the price is more than 26000 euro, but he can go down to 22000 euro to a lesser degree of satisfaction
5. If the color is black then the car has air conditioning
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The Language: Informally

We consider

- A fuzzy variant of $\mathcal{SHIF}$ with concrete data types
  - *e.g.*, $Sedan \sqcap (\geqslant price\ 22.000)$
- fuzzy constraints
  - numerical features are constrained by so-called fuzzy membership functions

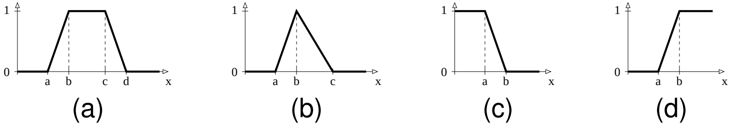

Figure: (a) Trapezoidal function $trz(a, b, c, d)$, (b) triangular function $tri(a, b, c)$, (c) left shoulder function $ls(a, b)$, and (d) right shoulder function $rs(a, b)$.

  - For instance, $(\exists price.ls(22000, 26000))$ dictates that given a price it returns the degree to which the constraint is satisfied

The Language

Definition (Concept expressions)

As for $\mathcal{SHIF}$

+

$$\begin{array}{rcl} C & \rightarrow & DR \quad (\text{datatype restriction}) \\ DR & \rightarrow & (\geqslant t\ val) \mid (\leqslant t\ val) \mid (= t\ val) \end{array}$$

e.g. $Sedan \sqcap (\leqslant price\ 26.000)$

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$$\begin{array}{rcl} C & \rightarrow & \forall t.d \mid \exists t.d \quad (\text{fuzzy constraints}) \\ d & \rightarrow & ls(a,b) \mid rs(a,b) \mid tri(a,b,c) \mid trz(a,b,c,d) \end{array}$$

e.g. $Car \sqcap (\exists price.ls(22000, 26000))$

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$$\begin{array}{rcl} C & \rightarrow & TC \quad (\text{threshold concept}) \\ TC & \rightarrow & C[\geqslant n] \mid C[\leqslant n] \end{array}$$

e.g. $(Sedan \sqcap Cheap \sqcap (\leqslant price\ 30.000))[\geqslant 0.8]$

+

$$\begin{array}{rcl} C & \rightarrow & WC \quad (\text{weighted sum concept}) \\ WC & \rightarrow & (w_1 \cdot C_1 + w_2 \cdot C_2 + \ldots + w_k \cdot C_k) \end{array}$$

where $\sum_{i=1}^{k} w_i = 1$. E.g., $0.2 \cdot (\leqslant price\ 30.000) + 0.8 \cdot (\exists hasColor.Red)$

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Semantics

- We need to give meaning to atomic constructs and logical connectives
- A *fuzzy interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is such that $\cdot^{\mathcal{I}}$ that assigns:
  1. to each abstract concept C a function $C^{\mathcal{I}} \colon \Delta^{\mathcal{I}} \to [0, 1]$
  2. to each abstract role R a function $R^{\mathcal{I}} \colon \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0, 1]$
  3. to each concrete role T a function $T^{\mathcal{I}} \colon \Delta^{\mathcal{I}} \times \Delta_D \to [0, 1]$

Semantics

- To extend $\cdot^{\mathcal{I}}$ to complex concepts, we need functions to define the negation, conjunction, disjunction (called norms), etc of values in $[0, 1]$
- The choice of them is not arbitrary, but is usually restricted: *e.g.*,

	Łukasiewicz Logic	Gödel Logic	Product Logic	Zadeh
$\ominus x$	$1 - x$	if $x = 0$ then 1 else 0	if $x = 0$ then 1 else 0	$1 - x$
$x \otimes y$	$\max(x + y - 1, 0)$	$\min(x, y)$	$x \cdot y$	$\min(x, y)$
$x \oplus y$	$\min(x + y, 1)$	$\max(x, y)$	$x + y - x \cdot y$	$\max(x, y)$
$x \Rightarrow y$	if $x \leqslant y$ then 1 else $1 - x + y$	if $x \leqslant y$ then 1 else y	if $x \leqslant y$ then 1 else y/x	$\max(1 - x, y)$

- Some salient properties

Property	Łukasiewicz Logic	Gödel Logic	Product Logic	Zadeh Logic
$x \otimes \ominus x = 0$	•	•	•	
$x \oplus \ominus x = 1$	•			
$x \otimes x = x$		•		•
$x \oplus x = x$		•		•
$\ominus \ominus x = x$	•			•
$x \Rightarrow y = \ominus x \oplus y$	•			•
$\ominus (x \Rightarrow y) = x \wedge \neg y$	•			•
$\ominus (x \otimes y) = \ominus x \oplus \ominus y$	•	•	•	•
$\ominus (x \oplus y) = \ominus x \otimes \ominus y$	•	•	•	•

- Note: if we want all properties, we collapse to boolean logic.

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- The mapping $\cdot^{\mathcal{I}}$ is extended complex concepts

$$
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\bot^{\mathcal{I}}(x) & = & 0 \\
\top^{\mathcal{I}}(x) & = & 1 \\
(\neg C)^{\mathcal{I}}(x) & = & \ominus C^{\mathcal{I}}(x)) \\
(C \sqcap D)^{\mathcal{I}}(x) & = & C^{\mathcal{I}}(x) \otimes D^{\mathcal{I}}(x) \\
(C \sqcup D)^{\mathcal{I}}(x) & = & C^{\mathcal{I}}(x) \oplus D^{\mathcal{I}}(x) \\
(\geqslant t\ val)^{\mathcal{I}}(x) & = & \sup_{c \in \Delta_D} t(x,v) \otimes (v \geqslant val)
\end{array}
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\begin{array}{lcl}
(= t\ val)^{\mathcal{I}}(x) & = & \sup_{c \in \Delta_D} t(x,v) \otimes (v = val) \\
(\forall R.C)^{\mathcal{I}}(x) & = & \inf_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \Rightarrow C^{\mathcal{I}}(y) \\
(\exists R.C)^{\mathcal{I}}(x) & = & \sup_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \otimes C^{\mathcal{I}}(y) \\
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(\leqslant t\ val)^{\mathcal{I}}(x) & = & \sup_{c \in \Delta_D} t(x,v) \otimes (v \leqslant val)
\end{array}
$$

$$
(w_1 C_1 + w_2 C_2 + \ldots + w_k C_k)^{\mathcal{I}}(x) = w_1 C_1{}^{\mathcal{I}}(x) + \cdots + w_k C_k{}^{\mathcal{I}}(x)
$$

$$
(C[\geqslant\ n])^{\mathcal{I}}(x) = \begin{cases} C^{\mathcal{I}}(x), & \text{if } C^{\mathcal{I}}(x) \geqslant n \\ 0, & \text{otherwise} \end{cases}
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(C[\leqslant\ n])^{\mathcal{I}}(x) = \begin{cases} C^{\mathcal{I}}(x), & \text{if } C^{\mathcal{I}}(x) \leqslant n \\ 0, & \text{otherwise} \end{cases}
$$

$$
(C \sqsubseteq D)^{\mathcal{I}} = \inf_{x \in \Delta^{\mathcal{I}}} C^{\mathcal{I}}(x) \Rightarrow D^{\mathcal{I}}(x)
$$

- Main inference task here:

given $\mathcal{K}$, determine the best satisfiability bound of a concept C:

$$
bsb(\mathcal{K}, C) = \sup_{\mathcal{I} \models \mathcal{K}} \sup_{x \in \Delta^{\mathcal{I}}} C^{\mathcal{I}}(x) .
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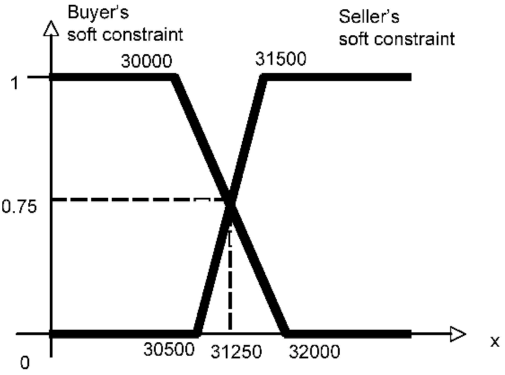
- Main inference task here:

given $\mathcal{K}$, determine the best satisfiability bound of a concept C:

$$bsb(\mathcal{K}, C) = \sup_{\mathcal{I} \models \mathcal{K}} \sup_{x \in \Delta^{\mathcal{I}}} C^{\mathcal{I}}(x) .$$



$$
\begin{aligned}
\mathit{Buy} &= \mathit{SportsCar} \sqcap \exists \mathit{price}.\mathit{ls}(30000, 32000) \\
\mathit{Sell} &= \mathit{SportsCar} \sqcap \exists \mathit{price}.\mathit{rs}(30500, 31500)
\end{aligned}
$$

$$
\mathit{bsb}(\mathcal{K}, \mathit{Buy} \sqcap \mathit{Sell}) = 0.75
$$

Hence, e.g., the car may be sold for 31250 euro.

Intended semantics

- We use Łukasiewicz Logic as the specific interpretation of the connectives
- The reasons for this choice:
  - nice logical (and computational) properties
  - $x \otimes_G y = \min(x, y) = x \otimes (x \Rightarrow y)$
  - $x \oplus_G y = \max(x, y) = \ominus(\ominus x \otimes_G \ominus y)$
  - Hence, we may use the macros:

$$
\begin{array}{lcl}
C \rightarrow D & := & \neg C \sqcup D \\
C \sqcap_G D & := & C \sqcap (C \rightarrow D) \\
C \sqcup_G D & := & \neg(\neg C \sqcap_G D))
\end{array}
$$

- More importantly, it guarantees Pareto optimality:

Theorem

If the maxima of $x \otimes y$, with $\langle x, y \rangle \in S \subseteq [0, 1] \times [0, 1]$, where $\otimes$ is Łukasiewicz t-norm, is positive then the maxima is also Pareto optimal

Note: also true for product t-norm, but not for Gödel t-norm $\otimes_G$

Problem Encoding

- Background knowledge: terminology $\mathcal{T}$ (GCI's may be fuzzy)

$$\mathcal{T} = \begin{cases} \mathit{Sedan} \sqsubseteq \mathit{PassengerCar} \\ \mathit{ExternalColorBlack} \sqsubseteq \neg \mathit{ExternalColorGray} \\ \mathit{SatelliteAlarm} \sqsubseteq \mathit{AlarmSystem} \\ \mathit{InsurancePlus} = \mathit{DriverInsurance} \sqcap \mathit{TheftInsurance} \\ \mathit{NavigatorPack} = \mathit{SatelliteAlarm} \sqcap \mathit{GPS\_system} \end{cases}$$

Hard Contraints

- The seller and the buyer model with concept definition σ and β the minimal requirements they accept for the negotiation
  - $\mathcal{T} \cup \{\sigma, \beta\}$ has to be satisfiable
  - possible agreement: interpretation $\mathcal{I}$ between β and σ such that $\mathcal{I} \models \mathcal{T} \cup \{\sigma, \beta\}$
  - Note: if $\mathcal{T} \cup \{\sigma, \beta\}$ has no models, then the negotiation ends immediately
- E.g., for the buyer, β is

$$B = (PassengerCar \sqcap (\leqslant price\ 30000))[\geqslant 1]$$

- E.g., for the seller, σ is

$$S = (Sedan \sqcap (\geqslant price\ 22000))[\geqslant 1]$$

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Soft Contraints

- The seller and the buyer model preferences with concept definitions β_i and σ_i (B_i and S_i are the defined concept names)
- The buyer's negotiation preference $\mathcal{B}$ is a concept definition of the form

$$PrefB = n_1 \cdot B_1 + \ldots + n_k \cdot B_k$$

- The seller's negotiation preference $\mathcal{S}$ is a concept definition of the form

$$PrefS = m_1 \cdot S_1 + \ldots + m_h \cdot S_h$$

- The weights n_i, m_j determine the importance of the preferences

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Example: Buyer's Soft Contraints

$$\beta_1: \; B_1 = \mathit{DriverInsurance} \sqcap (\mathit{TheftInsurance} \sqcup \mathit{FireInsurance})$$

$$\beta_2: \; B_2 = \mathit{AirConditioning} \sqcap (\mathit{ExternalColorBlack} \sqcup \mathit{ExternalColorGray})$$

$$\beta_3: \; B_3 = \exists \mathit{price}.\mathit{ls}(25000, 30000)$$

$$\beta_4: \; B_4 = \exists \mathit{km\_warranty}.\mathit{rs}(120000, 180000)$$

$$\mathcal{B}: \; \mathit{PrefB} = (0.05 \cdot B_1 + 0.05 \cdot B_2 + 0.8 \cdot B_3 + 0.1 \cdot B_4)$$

Example: Seller's Soft Contraints

σ_1: $S_1 = \mathit{InsurancePlus}$

σ_2: $S_2 = \exists \mathit{km\_warranty}.\mathit{ls}(140000, 175000)$

σ_3: $S_3 = \exists \mathit{price}.\mathit{rs}(22000, 26000)$

σ_4: $S_4 = \mathit{ExternalColorBlack} \sqcap \mathit{AirConditioning}$

$\mathcal{S}$: $\mathit{PrefS} = (0.05 \cdot S_1 + 0.3 \cdot S_2 + 0.6 \cdot S_3 + 0.05 \cdot S_4)$

Pareto Agreements

- Given
  - background theory $\mathcal{T}$
  - Buyer's hard constraint β
  - Buyer's negotiation preference $\mathcal{B}$: $PrefB = n_1 \cdot B_1 + \ldots + n_k \cdot B_k$
  - Seller's hard constraint σ
  - Seller's negotiation preference $\mathcal{S}$: $PrefS = m_1 \cdot S_1 + \ldots + m_h \cdot S_h$
- Consider $\bar{\mathcal{K}}$, where
$$\begin{aligned}\bar{\mathcal{T}} := \; & \mathcal{T} \cup \{\beta, \sigma\} \cup \bigcup_i \{\beta_i\} \cup \bigcup_j \{\sigma_j\} \\ & \cup \{Buy = B \sqcap PrefB[\geqslant t_b]\} \\ & \cup \{Sell = S \sqcap PrefS[\geqslant t_s]\} \\ & \cup \{Match = Buy \sqcap Sell\}\end{aligned}$$
- Pareto agreement value v_P:
$$v_P = bsb(\bar{\mathcal{K}}, Match)$$
- Pareto agreement: model $\mathcal{I}$ of $\bar{K}$ such that
$$v_P = \sup_{x \in \Delta^{\mathcal{I}}} (Buy \sqcap Sell)^{\mathcal{I}}(x) > 0$$
that is the *Pareto agreement value* is attained at $\mathcal{I}$ and has to be positive.
- Note that:
  - Pareto agreement value v_P is unique and optimal
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Running Example

- Pareto Agreement value: $v_P := 0.783333$
- Pareto Agreement: $\mathcal{I}$

$km\_warranty = 140000$,
$price = 25000$

Computing Optimal Agreements

- Can we compute the Pareto value and an agreement (*i.e.*, v_P and I)? Yes
- Solution: combination of logical inference + Mixed Integer Linear Programming
- Task supported by the *fuzzyDL* system

$$
\begin{aligned}
bsb(\mathcal{K}, C) &= \sup_{\mathcal{I}} \sup_{x \in \Delta^{\mathcal{I}}} \{C^{\mathcal{I}}(x) \mid \mathcal{I} \models \mathcal{K}\} \\
&= \max x \in [0,1] \\
&\qquad\qquad MILP(\mathcal{K} \cup \{a : C \geqslant x\})
\end{aligned}
$$

Conclusions and future work

- We have presented a logic-based approach for finding Pareto optimal agreements among Agents: it allows
  - to handle numerical features
  - to handle non numerical features
  - to deal with vagueness in agents preferences
- Future work:
  - experimentation
  - investigation of negotiation protocols, where an agreement is reachable in a reasonable amount of communication rounds

THANKS :-)