An OWL Ontology for Fuzzy OWL 2

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Abstract. The need to deal with vague information in Semantic Web languages is rising in importance and, thus, calls for a standard way to represent such information. We may address this issue by either extending current Semantic Web languages to cope with vagueness, or by providing an ontology describing how to represent such information within Semantic Web languages. In this work, we follow the latter approach and propose and discuss an OWL ontology to represent important features of fuzzy OWL 2 statements.

1 Introduction

It is well-known that "classical" ontology languages are not appropriate to deal with *fuzzy/vague/imprecise knowledge*, which is inherent to several real world domains. Since fuzzy set theory and fuzzy logic [24] are suitable formalisms to handle these types of knowledge, fuzzy ontologies emerge as useful in several applications, such as (multimedia) information retrieval, image interpretation, ontology mapping, matchmaking and the Semantic Web.

Description Logics (DLs) are the basis of several ontology languages. The current standard for ontology representation is OWL (Web Ontology Language), which comprises three sublanguages (OWL Lite, OWL DL and OWL Full). OWL 2 is a recent extension which is currently being considered for standardization [10]. The logical counterparts of OWL Lite, OWL DL and OWL 2 are the DLs $\mathcal{SHIF}(\mathbf{D})$, $\mathcal{SHOIN}(\mathbf{D})$, and $\mathcal{SROIQ}(\mathbf{D})$, respectively. OWL Full does not correspond to any DL, and reasoning with it is undecidable.

Several fuzzy extensions of DLs can be found in the literature (see the survey in [15]) and some fuzzy DL reasoners have been implemented, such as fuzzyDL [8], DeLorean [4] or Fire [16]. Not surprisingly, each reasoner uses its own fuzzy DL language for representing fuzzy ontologies and, thus, there is a need for a standard way to represent such information. We may address this issue by either extending current Semantic Web languages to cope with vagueness, or by providing an ontology describing how to represent such information within current Semantic Web languages.

In this work, we follow the latter approach and propose and discuss an OWL ontology to represent some important features of fuzzy OWL 2 statements. We have also developed two open-source parsers that map fuzzy OWL 2 statements

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expressed via this ontology into FUZZYDL and DELOREAN statements, respectively. Some appealing advantages of such an approach are that: *(i)* fuzzy OWL ontologies may easily be shared and reused according to the specified encoding; *(ii)* the ontology could easily be extended to include other types of fuzzy OWL 2 statements; *(iii)* current OWL editors can be used to encode a fuzzy ontology; and *(iv)* it can easily be translated into the syntax of other fuzzy DL reasoners.

The remainder of this paper is organized as follows. In Section 2 we present the definition of the DL $\mathcal{SROIQ}(\mathbf{D})$, the logic behind OWL 2, with fuzzy semantics. We also provide additional constructs, peculiar to fuzzy logic. Section 3 describes our OWL ontology, whereas Section 4 presents how to use it to represent two particular languages, those of FUZZYDL and DELOREAN fuzzy DL reasoners. Section 5 compares our proposal with the related work. Finally, Section 6 sets out some conclusions and ideas for future research.

2 The Fuzzy DL $\mathcal{SROIQ}$(D)

In this section we describe the fuzzy DL $\mathcal{SROIQ}(\mathbf{D})$, inspired by the logics presented in [3,8,21]. In the following, we assume $\bowtie \in \{\geq, >, \leq, <\}$, $\rhd \in \{\geq, >\}$, $\lhd \in \{\leq, <\}$, $\alpha \in (0, 1]$, $\beta \in [0, 1)$, $\gamma \in [0, 1]$.

Syntax. Similarly as for its crisp counterpart, fuzzy $\mathcal{SROIQ}(\mathbf{D})$ assumes three alphabets of symbols, for concepts, roles and individuals. Apart from atomic concept and roles, complex concept and roles can be inductively built.

Let us introduce some notation. C, D are (possibly complex) concepts, A is an atomic concept, R is a (possibly complex) abstract role, R_A is an atomic role, S is a simple role[1], T is a concrete role, $a, b \in \Delta^{\mathcal{I}}$ are *abstract individuals* and $v \in \Delta_{\mathbf{D}}$ is a *concrete individual*.

The syntax of fuzzy concepts and roles is shown in Table 1. Note that the syntax extends the crisp case with salient features of fuzzy DLs [3,8]: fuzzy nominals $\{\alpha_1/o_1, \ldots, \alpha_m/o_m\}$, fuzzy implication concepts $C \rightarrow D$, fuzzy weighted sums $\alpha_1 C_1 + \cdots + \alpha_k C_k$, modified concept and roles $mod(C)$ and $mod(R)$, cut concept and roles $[C \rhd \alpha]$ and $[R \rhd \alpha]$, and fuzzy datatypes $\mathbf{d}$. Furthermore, for each of the connectives $\sqcap, \sqcup, \rightarrow$, we allow the connectives $\sqcap_X, \sqcup_X, \rightarrow_X$, where $X \in$ {Gödel, Łukasiewicz, Product}, which are interpreted according to the semantics of the subscript.

As fuzzy concrete predicates we allow the following functions defined over $[k_1, k_2] \subseteq \mathbb{Q}^+ \cup \{0\}$: trapezoidal membership function (Fig. 1 (a)), the triangular (Fig. 1 (b)), the L-function (left-shoulder function, Fig. 1 (c)) and the R-function (right-shoulder function, Fig. 1 (d)) [20]. For instance, we may define $\mathsf{Young} \colon \mathbb{N} \rightarrow [0, 1]$, denoting the degree of a person being young, as $\mathsf{Young}(x) = L(10, 30)$. We also allow crisp intervals for backwards compatibility.

We further allow fuzzy modifiers, such as very. They are functions $f_m \colon [0, 1] \rightarrow [0, 1]$ which apply to fuzzy sets to change their membership function. We allow

[1] Simple roles are needed to guarantee the decidability of the logic. Intuitively, simple roles cannot take part in cyclic role inclusion axioms (see [7] for a formal definition).

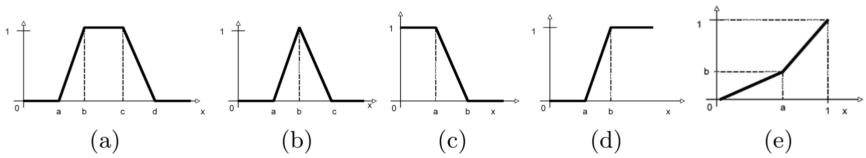

Fig. 1. (a) Trapezoidal function; (b) Triangular function; (c) L-function; (d) R-function; (f) Linear function

modifiers defined in terms of linear hedges (Fig. 1 (e)) and triangular functions (Fig. 1 (b)) [20]. For instance, $\mathsf{very}(x) = linear(0.8)$.

Example 1. Concept $\mathsf{Human} \sqcap \exists \mathsf{hasAge}.L(10, 30)$ denotes the set of young humans, with an age given by $L(10, 30)$. If $linear(4)$ represents the modifier very, $Human \sqcap linear(4)(\exists \mathsf{hasAge}.L(10, 30))$ denotes *very* young humans. □

A *Fuzzy Knowledge Base* (KB) contains a finite set of axioms organized in three parts: a fuzzy ABox $\mathcal{A}$ (axioms about individuals), a fuzzy TBox $\mathcal{T}$ (axioms about concepts) and a fuzzy RBox $\mathcal{R}$ (axioms about roles). A *fuzzy axiom* is an axiom that has a truth degree in [0,1]. The axioms that are allowed in our logic are: $\langle a : C \bowtie \gamma \rangle$, $\langle (a, b) : R \bowtie \gamma \rangle$, $\langle (a, b) : \neg R \bowtie \gamma \rangle$, $\langle (a, v) : T \bowtie \gamma \rangle$, $\langle (a, v) : \neg T \bowtie \gamma \rangle$, $\langle a \neq b \rangle$, $\langle a = b \rangle$, $\langle C \sqsubseteq D \rhd \gamma \rangle$, $\langle R_1 \dots R_m \sqsubseteq R \rhd \gamma \rangle$, $\langle T_1 \sqsubseteq T_2 \rhd \gamma \rangle$, $\mathtt{trans}(R)$, $\mathtt{dis}(S_1, S_2)$, $\mathtt{dis}(T_1, T_2)$, $\mathtt{ref}(R)$, $\mathtt{irr}(S)$, $\mathtt{sym}(R)$, and $\mathtt{asy}(S)$.

Example 2. $\langle \mathsf{paul} \colon \mathsf{Tall} \geq 0.5 \rangle$ states that Paul is tall with at least degree 0.5. The fuzzy RIA $\langle \mathsf{isFriendOf\ isFriendOf} \sqsubseteq \mathsf{isFriendOf} \geq 0.75 \rangle$ states that the friends of my friends can also be considered my friends with degree 0.75. □

Semantics. A fuzzy interpretation $\mathcal{I}$ with respect to a fuzzy concrete domain $\mathbf{D}$ is a pair $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consisting of a non empty set $\Delta^{\mathcal{I}}$ (the interpretation domain) disjoint with $\Delta_{\mathbf{D}}$ and a fuzzy interpretation function $\cdot^{\mathcal{I}}$ mapping:

- an *abstract individual* a onto an element $a^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$;
- a *concrete individual* v onto an element $v_{\mathbf{D}}$ of $\Delta_{\mathbf{D}}$;
- a *concept* C onto a function $C^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0, 1]$;
- an *abstract role* R onto a function $R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0, 1]$;
- a *concrete role* T onto a function $T^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta_{\mathbf{D}} \to [0, 1]$;
- an n-ary *concrete fuzzy predicate* $\mathbf{d}$ onto the fuzzy relation $\mathbf{d}_{\mathbf{D}} : \Delta^n_{\mathbf{D}} \to [0, 1]$;
- a *modifier mod* onto a function $f_{mod} : [0, 1] \to [0, 1]$.

Given arbitraries t-norm $\otimes$, t-conorm $\oplus$, negation function $\ominus$ and implication function $\Rightarrow$ (see [13] for properties and examples of these fuzzy operators), the fuzzy interpretation function is extended to fuzzy *complex concepts, roles* and *axioms* as shown in Table 1.

$C^{\mathcal{I}}$ denotes the membership function of the fuzzy concept C with respect to the fuzzy interpretation $\mathcal{I}$. $C^{\mathcal{I}}(x)$ gives us the degree of being the individual x an element of the fuzzy concept C under $\mathcal{I}$. Similarly, $R^{\mathcal{I}}$ denotes the membership

Table 1. Syntax and semantics of concepts, roles and axioms in fuzzy $\mathcal{SROIQ}(\mathbf{D})$

Syntax (concept C)	**Semantics of $C^{\mathcal{I}}(x)$**
$\top$	1
$\bot$	0
A	$A^{\mathcal{I}}(x)$
$C \sqcap D$	$C^{\mathcal{I}}(x) \otimes D^{\mathcal{I}}(x)$
$C \sqcup D$	$C^{\mathcal{I}}(x) \oplus D^{\mathcal{I}}(x)$
$\neg C$	$\ominus C^{\mathcal{I}}(x)$
$\forall R.C$	$\inf_{y \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(x,y) \Rightarrow C^{\mathcal{I}}(y)\}$
$\exists R.C$	$\sup_{y \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(x,y) \otimes C^{\mathcal{I}}(y)\}$
$\forall T.\mathbf{d}$	$\inf_{v \in \Delta_{\mathbf{D}}} \{T^{\mathcal{I}}(x,v) \Rightarrow \mathbf{d}_{\mathbf{D}}(v)\}$
$\exists T.\mathbf{d}$	$\sup_{v \in \Delta_{\mathbf{D}}} \{T^{\mathcal{I}}(x,v) \otimes \mathbf{d}_{\mathbf{D}}(v)\}$
$\{\alpha_1/o_1, \ldots, \alpha_m/o_m\}$	$\sup_{\{\, i \mid x = o_i^{\mathcal{I}} \}} \alpha_i$
$\geq m\ S.C$	$\sup_{y_1,\ldots,y_m \in \Delta^{\mathcal{I}}} [(\min_{i=1}^{m} \{S^{\mathcal{I}}(x,y_i) \otimes C^{\mathcal{I}}(y_i)\}) \bigotimes (\otimes_{j<k} \{y_j \neq y_k\})]$
$\leq n\ S.C$	$\inf_{y_1,\ldots,y_{n+1} \in \Delta^{\mathcal{I}}} [(\min_{i=1}^{n+1} \{S^{\mathcal{I}}(x,y_i) \otimes C^{\mathcal{I}}(y_i)\}) \Rightarrow (\oplus_{j<k} \{y_j = y_k\})]$
$\geq m\ T.\mathbf{d}$	$\sup_{v_1,\ldots,v_m \in \Delta_{\mathbf{D}}} [(\min_{i=1}^{m} \{T^{\mathcal{I}}(x,v_i) \otimes \mathbf{d}_{\mathbf{D}}(v_i)\}) \bigotimes (\otimes_{j<k} \{v_j \neq v_k\})]$
$\leq n\ T.\mathbf{d}$	$\inf_{v_1,\ldots,v_{n+1} \in \Delta_{\mathbf{D}}} [(\min_{i=1}^{n+1} \{T^{\mathcal{I}}(x,v_i) \otimes \mathbf{d}_{\mathbf{D}}(v_i)\}) \Rightarrow (\oplus_{j<k} \{v_j = v_k\})]$
$\exists S.Self$	$S^{\mathcal{I}}(x,x)$
$mod(C)$	$f_{mod}(C^{\mathcal{I}}(x))$
$[C \geq \alpha]$	1 if $C^{\mathcal{I}}(x) \geq \alpha$, 0 otherwise
$[C \leq \beta]$	1 if $C^{\mathcal{I}}(x) \leq \beta$, 0 otherwise
$\alpha_1 C_1 + \cdots + \alpha_k C_k$	$\alpha_1 {C_1}^{\mathcal{I}}(x) + \cdots + \alpha_k {C_k}^{\mathcal{I}}(x)$
$C \rightarrow D$	$C^{\mathcal{I}}(x) \Rightarrow D^{\mathcal{I}}(x)$
Syntax (role R)	**Semantics of $R^{\mathcal{I}}(x,y)$**
R_A	$R_A^{\mathcal{I}}(x,y)$
U	1
R^-	$R^{\mathcal{I}}(y,x)$
$mod(R)$	$f_{mod}(R^{\mathcal{I}}(x,y))$
$[R \geq \alpha]$	1 if $R^{\mathcal{I}}(x,y) \geq \alpha$, 0 otherwise
T	$T^{\mathcal{I}}(x,v)$
Syntax (axiom τ)	**Semantics of $\tau^{\mathcal{I}}$**
$a : C$	$C^{\mathcal{I}}(a^{\mathcal{I}})$
$(a,b) : R$	$R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}})$
$(a,b) : \neg R$	$\ominus R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}})$
$(a,v) : T$	$T^{\mathcal{I}}(a^{\mathcal{I}}, v_{\mathbf{D}})$
$(a,v) : \neg T$	$\ominus T^{\mathcal{I}}(a^{\mathcal{I}}, v_{\mathbf{D}})$
$C \sqsubseteq D$	$\inf_{x \in \Delta^{\mathcal{I}}} C^{\mathcal{I}}(x) \Rightarrow D^{\mathcal{I}}(x)$
$R_1 \ldots R_m \sqsubseteq R$	$\inf_{x_1, x_{n+1}} \sup_{x_2,\ldots,x_n} (R_1^{\mathcal{I}}(x_1,x_2) \otimes \cdots \otimes R_n^{\mathcal{I}}(x_n,x_{n+1})) \Rightarrow R^{\mathcal{I}}(x_1,x_{n+1})$ where $x_1 \ldots x_{n+1} \in \Delta^{\mathcal{I}}$
$T_1 \sqsubseteq T_2$	$\inf_{x \in \Delta^{\mathcal{I}}, v \in \Delta_{\mathbf{D}}} T_1^{\mathcal{I}}(x,v) \Rightarrow T_2^{\mathcal{I}}(x,v)$

function of the fuzzy role R with respect to $\mathcal{I}$. $R^{\mathcal{I}}(x,y)$ gives us the degree of being (x,y) an element of the fuzzy role R under $\mathcal{I}$.

Let $\phi \in \{a : C, (a,b) : R, (a,b) : \neg R, (a,v) : T, (a,v) : \neg T\}$ and $\psi \in \{C \sqsubseteq D, R_1 \ldots R_m \sqsubseteq R, T_1 \sqsubseteq T_2\}$. $\phi^{\mathcal{I}}$ and $\psi^{\mathcal{I}}$ are defined in Table 1. Then, a fuzzy interpretation $\mathcal{I}$ *satisfies* (is a *model* of):

- $\langle \phi \bowtie \gamma \rangle$ iff $\phi^{\mathcal{I}} \bowtie \gamma$,
- $\langle a \neq b \rangle$ iff $a^{\mathcal{I}} \neq b^{\mathcal{I}}$,
- $\langle a = b \rangle$ iff $a^{\mathcal{I}} = b^{\mathcal{I}}$,
- $\langle \psi \rhd \gamma \rangle$ iff $\psi^{\mathcal{I}} \rhd \gamma$,
- $\mathtt{trans}(R)$ iff $\forall x, y \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(x,y) \geq \sup_{z \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,z) \otimes R^{\mathcal{I}}(z,y)$,
- $\mathtt{dis}(S_1, S_2)$ iff $\forall x, y \in \Delta^{\mathcal{I}}, S_1^{\mathcal{I}}(x,y) = 0$ or $S_2^{\mathcal{I}}(x,y) = 0$,
- $\mathtt{dis}(T_1, T_2)$ iff $\forall x \in \Delta^{\mathcal{I}}, v \in \Delta_{\mathbf{D}}, T_1^{\mathcal{I}}(x,v) = 0$ or $T_2^{\mathcal{I}}(x,v) = 0$,
- $\mathtt{ref}(R)$ iff $\forall x \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(x,x) = 1$,
- $\mathtt{irr}(S)$ iff $\forall x \in \Delta^{\mathcal{I}}, S^{\mathcal{I}}(x,x) = 0$,
- $\mathtt{sym}(R)$ iff $\forall x, y \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(x,y) = R^{\mathcal{I}}(y,x)$,

– $\mathsf{asy}(S)$ iff $\forall x, y \in \Delta^{\mathcal{I}}$, if $S^{\mathcal{I}}(x, y) > 0$ then $S^{\mathcal{I}}(y, x) = 0$,
– a fuzzy KB iff it satisfies each of its axioms.

3 An OWL Ontology for Fuzzy OWL 2

In this section we describe FuzzyOwl2Ontology, the OWL ontology that we have developed with the aim of representing a fuzzy extension of OWL 2. An excerpt of the ontology is shown in Fig. 2.

FuzzyOwl2Ontology has 8 main classes representing different elements of a fuzzy ontology (of course, each of these classes has several subclasses):

– Individual simply represents an individual of the vocabulary.
– Concept, represents a fuzzy concept of the vocabulary. A concept can be an AbstractConcept or a ConcreteConcept. These two classes have several subclasses, covering the complex constructors already defined in Section 2.
– Property, represents a fuzzy role. A property can be concrete (DatatypeProperty) or abstract (ObjectProperty). These two classes have a lot of subclasses, covering the complex constructors already defined in Section 2.
– Axiom represents the axioms defined in Section 2. Axioms can be grouped in three categories: ABoxAxiom, TBoxAxiom and RBoxAxiom. Some of the

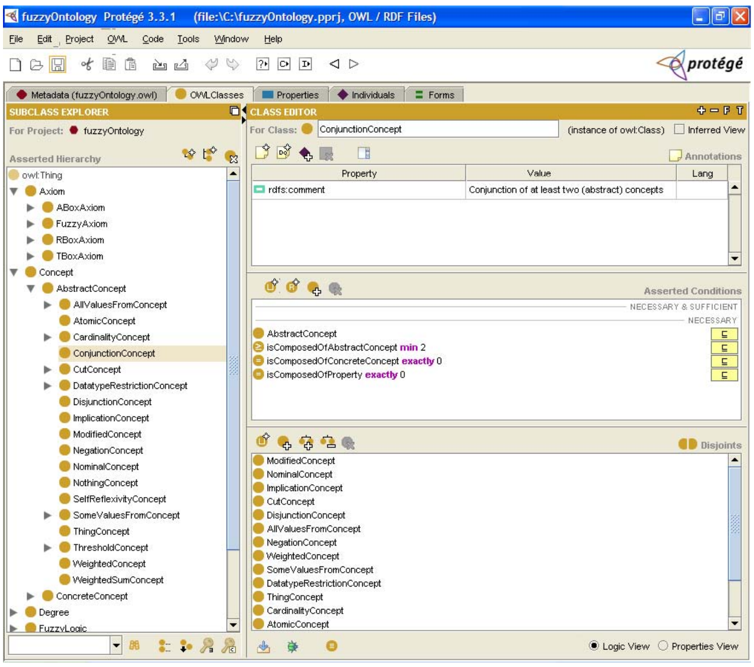

Fig. 2. An excerpt of FuzzyOwl2Ontology

axioms are subclasses of FuzzyAxiom, which indicates that the axiom is not either true or false, but that it is true to some extent.
- Degree represents a degree which can be added to an instance of FuzzyAxiom. LinguisticDegree, ModifierDegree, NumericDegree and Variable are subclasses.
- Query represents a special kind of axioms, queries to be submitted to a fuzzy DL reasoner. Current subclasses are ConceptSatisfiabilityQuery, EntailmentQuery, GreatestConceptSatisfiabilityQuery, GreatestLowerBoundQuery, LowestUpperBoundQuery, OntologyConsistencyQuery and SubsumptionQuery.
- FuzzyLogic represents different families of fuzzy operators which can be used to give different semantics to the logic. Current subclasses are Zadeh, Goedel, Lukasiewicz and Product. They can be linked via the property hasSemantics.
- FuzzyModifier represents a fuzzy modifier which can be used to modify the membership function of a fuzzy concept or a fuzzy role. Current subclasses are LinearFuzzyModifier and TriangularFuzzyModifier.

There are also some object properties, establishing relations between concepts, and datatype properties, defining attributes of them.

Example 3

- A ConceptAssertion axiom has several object properties: isComposedOfAbstractConcept, isComposedOfAbstractIndividual and hasDegree.
- A TriangularConcreteFuzzyConcept has several datatype properties representing the parameters k_1, k_2, a, b, c of the membership function, as shown in Section 2: hasParameterA, hasParameterB, hasParameterC, hasParameterK1 and hasParameterK2. □

The integrity of the semantics is maintained with several domain, range, functionality and cardinality axioms.

Example 4

- The range of isComposedOfIndividual is Individual.
- ConceptAssertion has exactly one relation isComposedOfAbstractIndividual.
- DisjointConceptAssertion has at least two isComposedOfConcept relations. □

In some cases the order of the relations is important. For example, ConceptInclusion is related to two concepts, one being the subsumer and another being the subsumed. A similar situation happens with PropertyAssertion and PropertyInclusion. For this reason, there are two types of isComposedOfConcept, isComposedOfAbstractIndividual and isComposedOfProperty axioms. In the special case of ComplexObjectPropertyInclusionAxiom the subsumed role is related (via hasChainProperty) to another role and so on, forming a chain of roles.

Currently, our ontology has 168 classes, 28 object properties, 11 datatype properties and no instances. When a user wants to build a fuzzy ontology using FuzzyOWL2Ontology, he needs to populate our ontology with instances representing the axioms and the elements of its ontology.

Example 5. In order to represent that car125 is an expensive Sedan car with at least degree 0.5, we proceed as follows:

- Create an instance of Individual called car125.
- Create an instance of AtomicConcept called Sedan.
- Create an instance of AtomicConcept called ExpensiveCar.
- Create an instance of ConjunctionConcept. Assume that the name is conj1.
- Create an instance of NumericDegree. Assume that the name is deg1.
- Create an instance of ConceptAssertion. Assume that the name is ass1.
- Create a datatype property hasNumericValue between deg1 and "0.5".
- Create an object property isComposedOfAbstractConcept between conj1 and Sedan, and another one between conj1 and ExpensiveCar.
- Create an object property isComposedOfAbstractIndividual between ass1 and car125, and another one between ass1 and conj1.
- Create an object property hasNumericDegree between ass1 and deg1. □

Once the user has populated the ontology, it is possible to perform a consistency test over the OWL ontology, in order to check that all the axioms (for example, functionality of the roles) are verified, and thus, that the fuzzy ontology is syntactically correct. It is not possible though to check if the fuzzy ontology is consistent using standard reasoners for OWL.

4 FuzzyOWL2Ontology in Use

As an example of application of the FuzzyOwl2Ontology, we have developed two open-source parsers mapping fuzzy ontologies represented using this ontology into the syntax supported by different fuzzy DL reasoners, in particular fuzzyDL [8] and DeLorean [4]. Currently, the parsers support fuzzy $\mathcal{SHIF}(\mathbf{D})$, the common fragment to them.

The syntax of fuzzyDL can be found in [8], whereas the syntax of DeLorean can be found in [2]. Both fuzzy DL reasoners have a similar Lisp-based syntax, but there are a lot of differences, which makes the manual codification of a fuzzy ontology in the two syntaxes a very tedious and error-prone task. This can be avoided by using FuzzyOwl2Ontology as an intermediate step.

The parsers have been developed in Java language using OWL API 2[2], which is an open-source API for manipulating OWL 2 ontologies [14] (we recall though that FuzzyOwl2Ontology is in OWL).

Each of these parsers works as follows (and consequently, similar parsers could be easily built). The input is a text file containing an ontology obtained after having populated FuzzyOwl2Ontology with OWL statements represented axioms in fuzzy $\mathcal{SROIQ}(\mathbf{D})$. To start with, OWL API is used to obtain a model representing the OWL ontology. Then, we iterate over the axioms and, for each of them, we compute the translation into the syntax of the particular fuzzy DL reasoner. We do not only have to translate the axioms, but also the elements (concepts, roles, individuals, fuzzy concrete domains ...) that take part in it.

Given an instance of the Axiom class, the parser navigates through its relation to obtain its components (for example, in a ConceptAssertion the parser gets the fillers for isComposedOfAbstractConcept, isComposedOfAbstractIndividual and hasDegree. A similar situation occurs with complex concepts and roles.

[2] http://owlapi.sourceforge.net

The parser also takes into account the fact that some of the axioms may need to introduce a previous definition. For instance, in FUZZYDL we need to define a trapezoidal fuzzy concept before using it.

FuzzyOwl2Ontology is very expressive, and no reasoner can currently support all of its constructors. Hence, if the reasoner does not support an OWL statement or one of the elements that take part in it, a warning message is shown and the axiom is skipped.

Example 6. As an example of the differences between them, assume that the age of a person ranges in $[0, 200]$ and consider the concept $\exists \mathsf{hasAge}.L(10; 30)$.

In DeLorean, it is represented as: (`some hasAge` (`trapezoidal` 0 10 30 200)).

In FUZZYDL, in addition to the axiom we also need a previous definition:

(`define-fuzzy-concept trap left-shoulder`(0, 200, 10, 30))
(`some hasAge trap`) □

In order to demonstrate the coverage of OWL 2, we have also developed a parser translating an OWL 2 ontology (for the moment without datatypes) into FuzzyOwl2Ontology. This way, the user can import existing ontologies in an automatic way, as a previous step to extend them to the fuzzy case.

5 Discussion and Related Work

This is, to the best of our knowledge, the first ontology for fuzzy ontology representation. However, a similar idea has been presented in [1], where an OWL ontology is used to describe and build fuzzy relational databases.

The W3C Uncertainty Reasoning for the World Wide Web Incubator Group (URW3-XG) defined an ontology of uncertainty, a vocabulary which can be used to annotate different pieces of information with different types of uncertainty (e.g. vagueness, randomness or incompleteness), the nature of the uncertainty, etc. [23]. But unlike our ontology, it can only be used to identify some kind of uncertainty, and not to represent and manage uncertain pieces of information.

Fuzzy extensions of ontology languages have been presented, more precisely OWL [11,18] and OWL 2 [17], but they are obviously not complaint with the current standard.

A pattern for uncertainty representation in ontologies has also been presented [22]. However, it relies in OWL Full, thus not making possible for instance to check the syntactic correctness of the fuzzy ontology.

Our approach should not be confused with a series of works that describe, given a fuzzy ontology, how to obtain an equivalent OWL ontology (see for example [2,3,5,6,7,9,17,19]). In these works it is possible to reason using a crisp DL reasoner instead of a fuzzy DL reasoner, which is not our case. However, the obtained OWL ontologies cannot be easily understood by humans, as it happens under our approach.

Another approach to represent uncertainty without extending the standard language is to use annotation properties [12]. Despite the simplicity of this approach, it also has several disadvantages with respect to our approach. The formal semantics of annotation properties is rather limited. More precisely, it is not possible to reason using standard tools with the fuzzy part of the ontology. The fact that an essential part of the ontology is not automatically understood is actually opposed to the philosophy of ontologies. Annotations are useful for "minimalist" extensions of the language, such as for example just adding a degree to an axiom. However, they are not so appropriate for new concept or role constructors. Furthermore, it uses OWL 2 annotation properties, whereas we are complaint with the current standard language OWL.

6 Conclusions and Future Work

In this paper we have proposed FuzzyOwl2Ontology, an OWL ontology to represent fuzzy extensions of the OWL and OWL 2 languages. The main advantages of our approach is that we are complaint with the standard ontology language and we can perform some reasoning with the meta-model by using standard OWL reasoners. We have proved its utility by means of a couple of parsers translating fuzzy ontologies represented with FuzzyOwl2Ontology into the common fragment of the languages supported by the fuzzy DL reasoners fuzzyDL and DeLorean. We have also implemented a parser translating from OWL 2 into FuzzyOwl2Ontology statements.

Our approach is extensible, the ontology can easily be augmented to other fuzzy statements, and similar parsers could be built for other fuzzy DL reasoners. The parsers and the ontology are available from the fuzzyDL web site[3].

In future work we plan to develop a graphical interface such as a Protégé plug-in to assist users in the population of FuzzyOwl2Ontology. We would also like to extend the parser to fully cover the languages supported by fuzzyDL and DeLorean, and to cover the opposite directions of the translations.

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